Analyzing Uncertainty and Optimizing: A Case Study in Retail

Alkis Vazacopoulos
Robert Ashford
Optimization Direct Inc.

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Summary

- Retailer buys a season's stock in advance
  - Tries to sell it over course of season
  - Uses ad-hoc mark-downs, promotions, etc.
  - Needs a quantitative way to manage these
- Model demand by analyzing past sales
- Optimize discounts
- Examine season length and increasing prices

The Retail Problem

- Budget apparel retailer with ~ 150 stores in ~30 states
- Buys stock for a season from Far East
  - 200 – 600 separate SKUs
  - Different styles, colors and sizes
- Stock quantity and price from suppliers agreed in advance
- Need to sell this stock over limited season
  - e.g. 24 weeks
- Unsold stock at end of season has little value
- Decide sale price and season length

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Revenue from a single SKU

- Define:

  Stock s
  Demand ξ
  Sale price p
  Residual value r

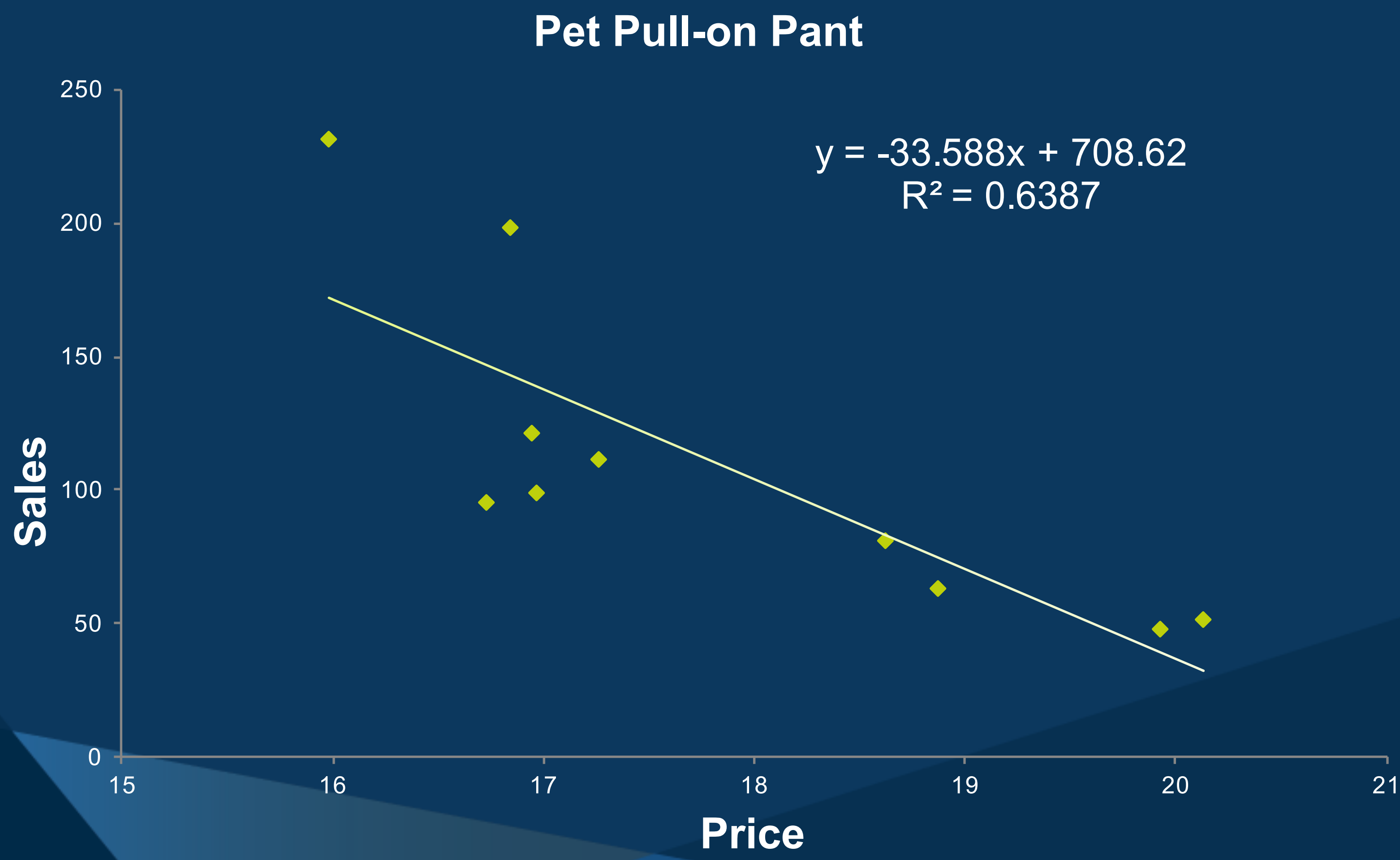
- Revenue is:

  $p\ \mathit{min}\{ s, \xi \} + r\ \mathit{max}\{ s - \xi, 0 \}$

Modeling Demand

- Consider demand in each week of season to be independent
  - Demand for remainder of season sum of independent weekly demands in remaining weeks
- Demand = sales if no stock out
- Mean demand depends on price
- Assume mean demand linear in price or log(price)
  - Although the slope β_1 **may be zero**

Price-Demand Elasticity



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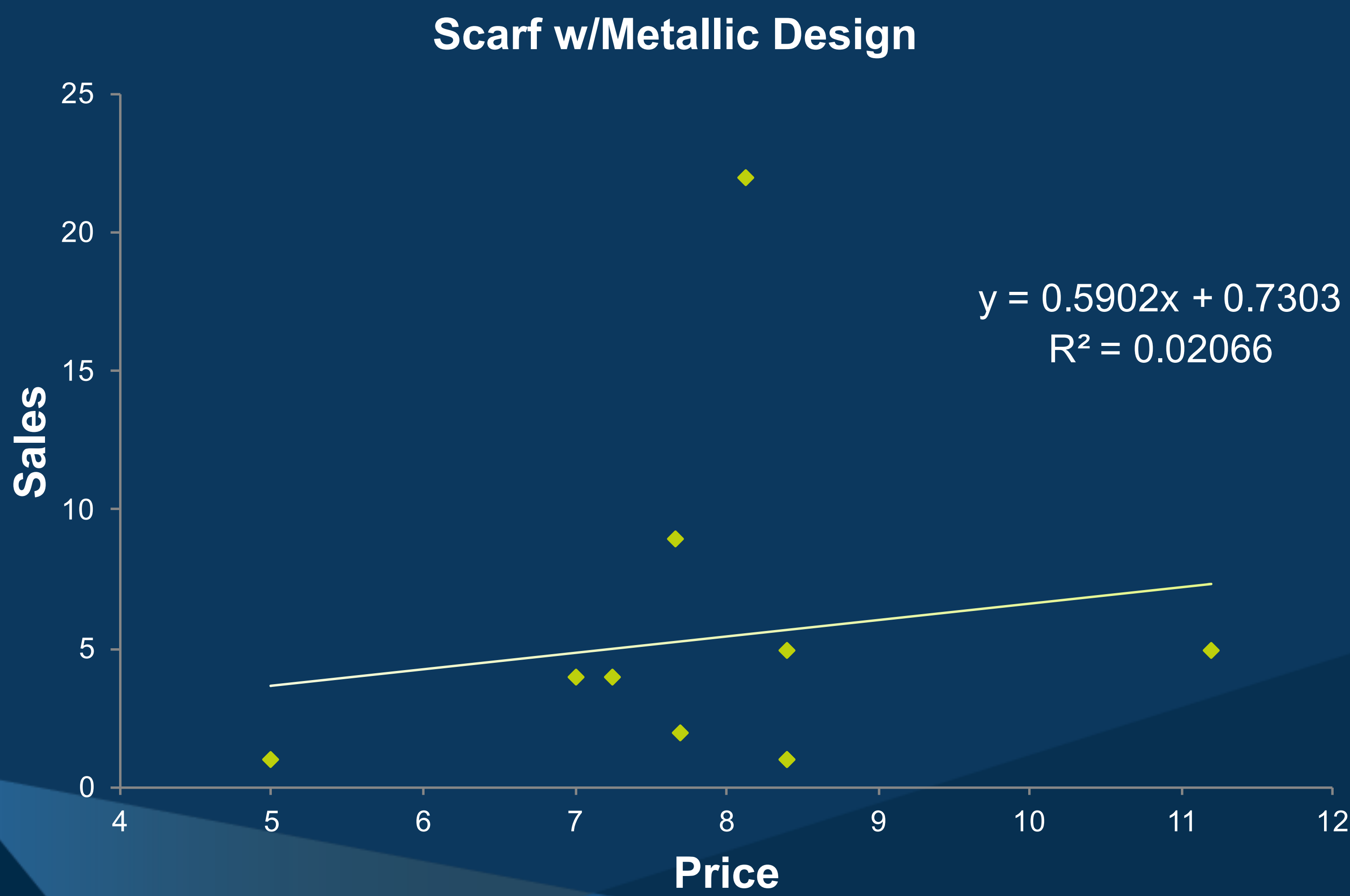
Price-Demand Elasticity



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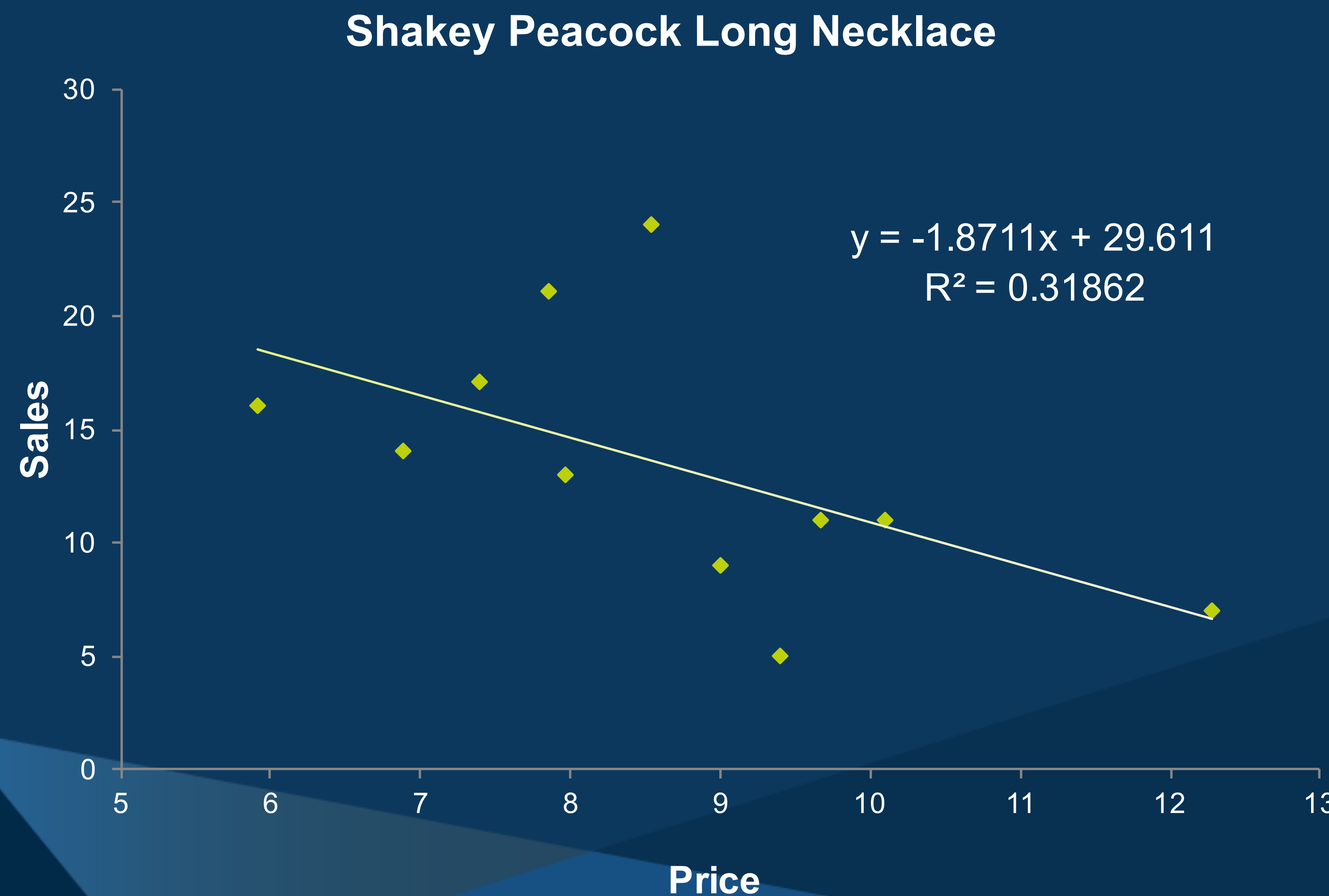

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Modeling Random Demand

- Suppose sales are a Poisson process
- Implies it is Binomially distributed
- Approximate with a Normal distribution
- Demand ξ distributed as $N(\mu, \sqrt{\mu})$
- In practice, shoppers buy more than a single item, so model

  $$\xi \sim N(\mu, \gamma \sqrt{\mu})$$

  where mean demand $\mu \equiv \mu(p)$
  and γ is a factor which differs for each SKU

- Demand or different SKUs in same period is NOT independent
  - Positive correlation on average

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Expected value and variance of revenue from single SKU

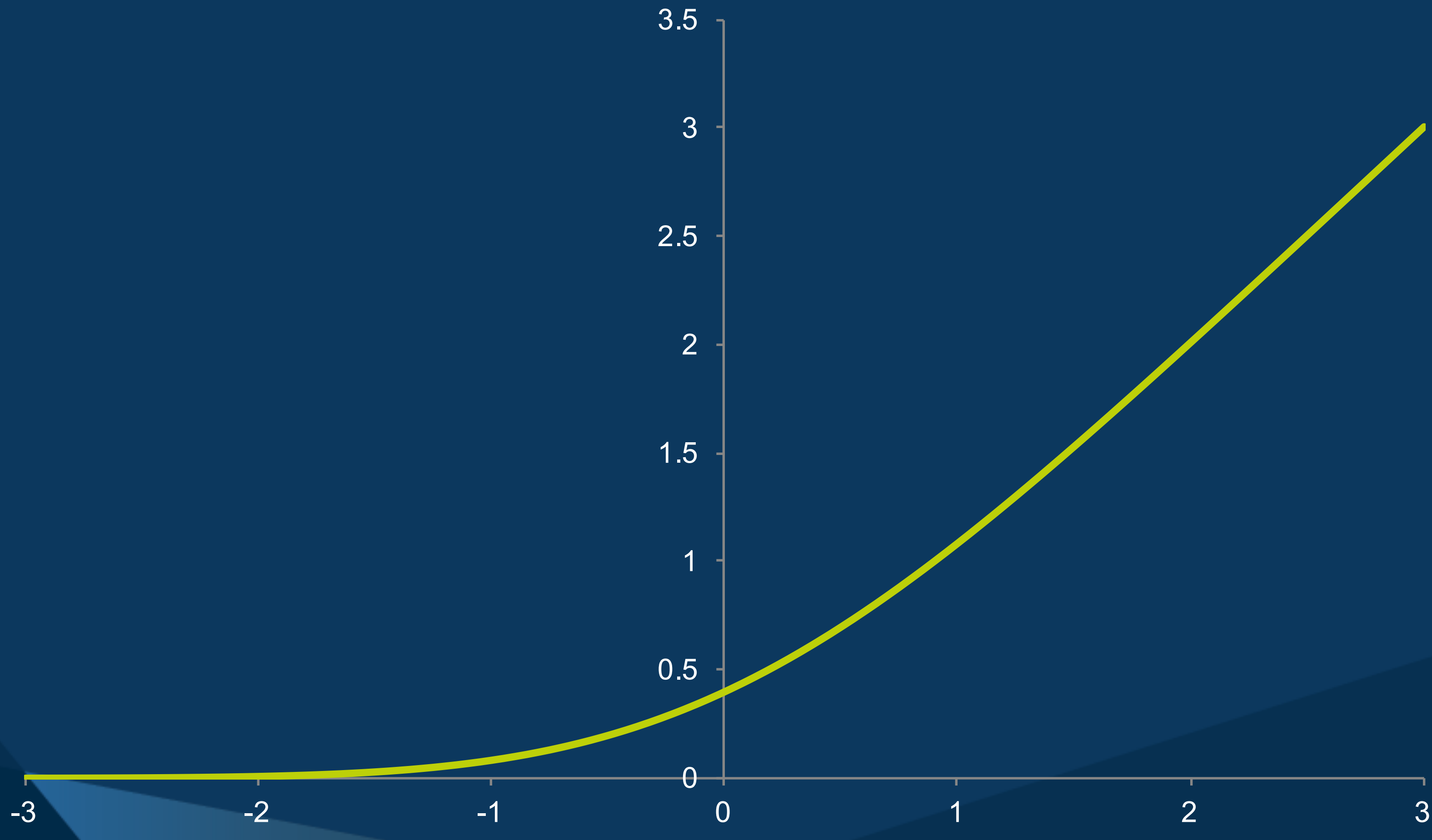
- Can calculate the expected value of revenue as:

$$p\,s - (p - r)\,\sigma\,\Psi((s-\mu)/\sigma)$$

where $\sigma \equiv \gamma\sqrt{\mu}$
and $\Psi(X) \equiv X\,\Phi(X) + \phi(X)$,
$\phi(X) = \Phi'(X)$, $\Phi(X)$ **being the Gaussian distribution fn.**

- Can calculate the variance of sales
  - More complicated expression

Ψ(X)



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Expected value of future revenue from single SKU

- Model demand as
- $\xi = \beta_0 + \beta_1 p + \varepsilon$
  where residuals ε have zero mean and
- mean demand $\mu(p) \equiv E[\xi] = \beta_0 + \beta_1 p$ which we estimate by
  $\hat{\mu}(p) = \hat{\beta}_0 + \hat{\beta}_1 p$
- Thus the expected value of revenue in future period may be estimated by
  $p\, s - (p - r)\, \hat{\sigma}(p)\, \Psi(\, (s-\hat{\mu}(p))/\hat{\sigma}(p)\,)$
  where
  $\hat{\sigma}(p)^2 = \hat{Var}(\xi) = Var(\hat{\mu}(p)) + \hat{\gamma}\, \sqrt{\hat{\mu}}(p)$

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Expected value of future revenue from all SKUs

- Add expected future revenue from individual SKUs, but
- Take care in computing σ for all future periods (ε's are independent)
- Use hetroscedastic regression to estimate $\hat{\gamma}$'s
- Use sample SKU correlations in computing variance of revenue
- Adjust approximate demand distribution functions to ban negative sales

Optimizing Price

- Can choose price as:
  - initial sales price
  - initial sales price less 10%
  - initial sales price less 20% ...
- Make best choice for each (group of) SKUs
- Use sales data in season so far
- Set prices at the start of each week
- Increase expected future revenues by 19% (+/- 6%)

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Allowing price increases

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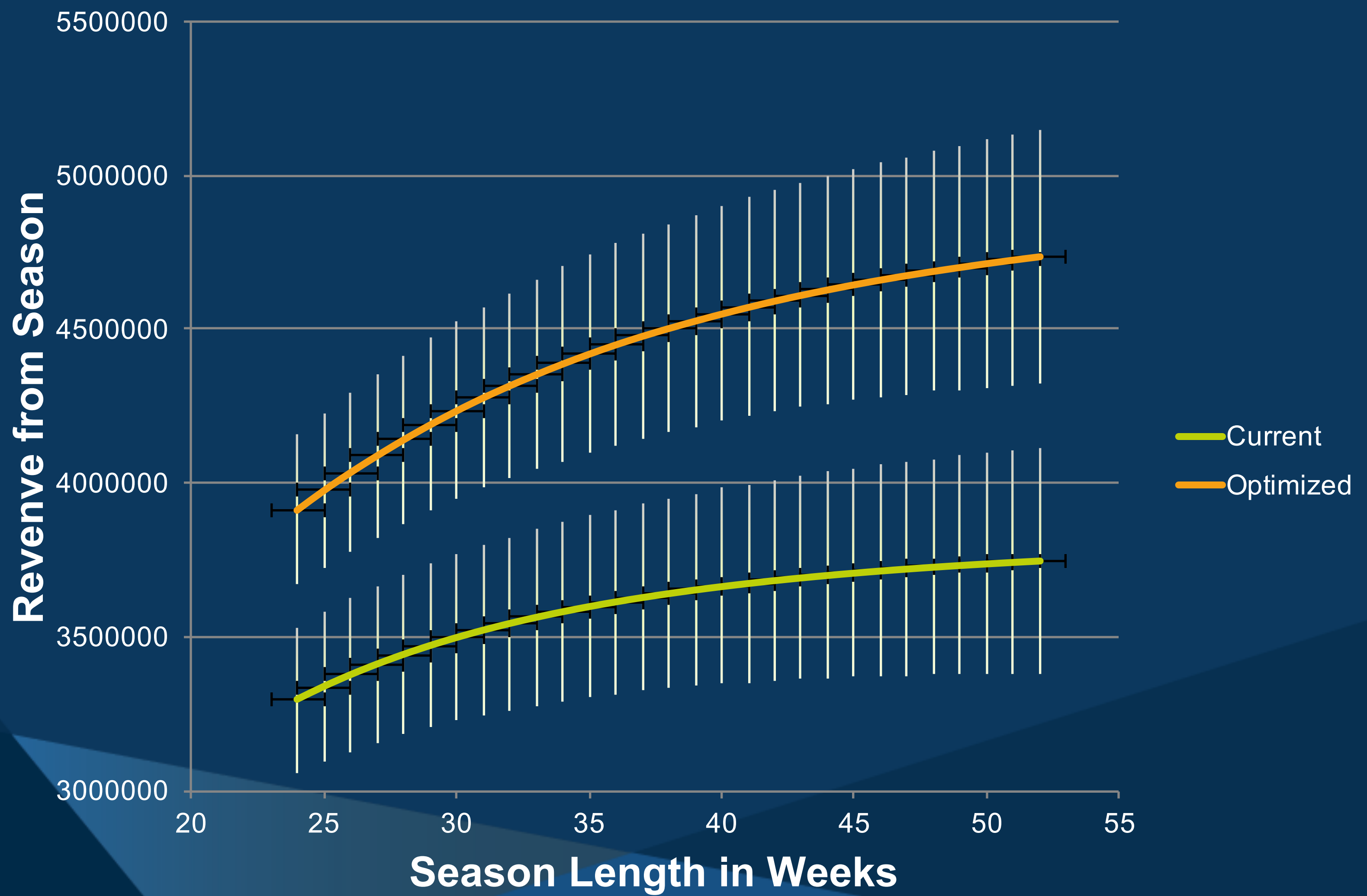
Longer season length



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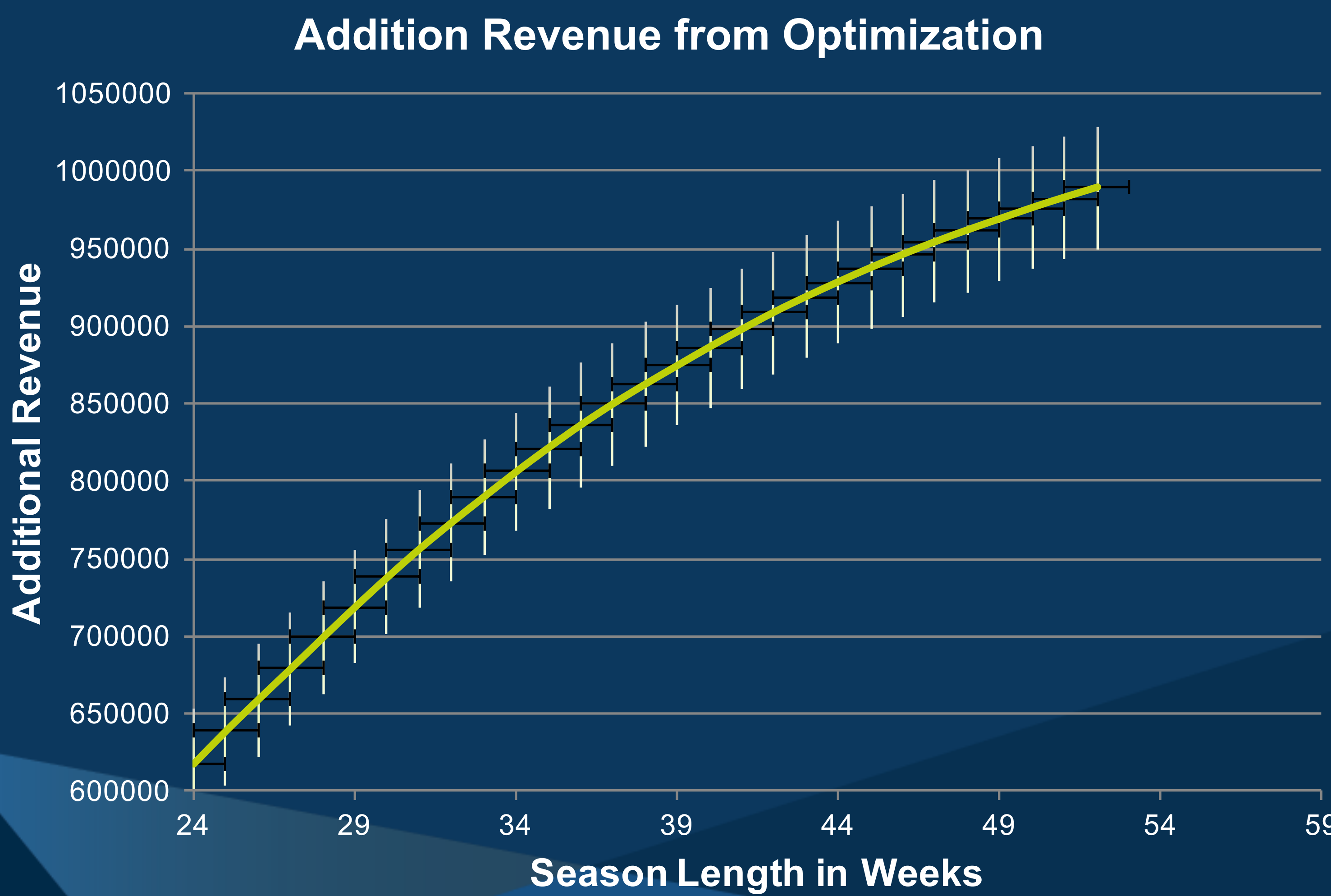
Longer season length



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Conclusions

- Help with analytical determination of discounts
- Asses effect of raising prices
- Evaluate likely cost of limited season length
- Must be mindful of underlying assumptions
  - Increasing price
  - Decline in demand over course of season

Thanks for listening

Robert Ashford
rwa@optimizationdirect.com
www.optimizationdirect.com

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