12.
Inventory Management

Outline

- The Importance of Inventory
- Inventory Costs
- ABC Analysis
- EOQ Models
- Probabilistic Models and Safety Stock
- Inventory Control Systems
- Single-Period Model
- Using Simulation for Inventory Management

What is Inventory?

- Stock of items kept to meet future demand for
  - internal customers
  - external customers
- Purpose of inventory management
  - ORDERING POLICY: When and how many units to order of each material when orders are placed with either outside suppliers or production departments within organizations?
  - ISSUING POLICY: how to issue units from inventory? (FIFO, LIFO, random?)

Importance of Inventory

- Inventories are important to all types of firms:
  - They have to be counted, paid for, used in operations, used to satisfy customers, and managed
  - Too much inventory reduces profitability
  - Too little inventory damages customer confidence
- It is one of the most expensive assets of many companies representing as much as 50% of total invested capital
- It is one of the 3 most common reasons for SME bankruptcy
- Need to balance inventory investment and customer service

Why Do We Want to Hold Inventory

- Improve customer service
- Safe-guard to hazards in demand, supply, and delivery that might cause stock-out
- Take advantage of economies of scale, & reduce:
  - ordering costs
  - Stock-out costs
  - acquisition costs
  - Fixed costs (e.g. fixed ordering costs)
- Contribute to the efficient and effective operation of the production system, e.g.,
  - Reduces the number of costly set-ups and reschedulings
  - Smoothing and stabilizing resource utilization

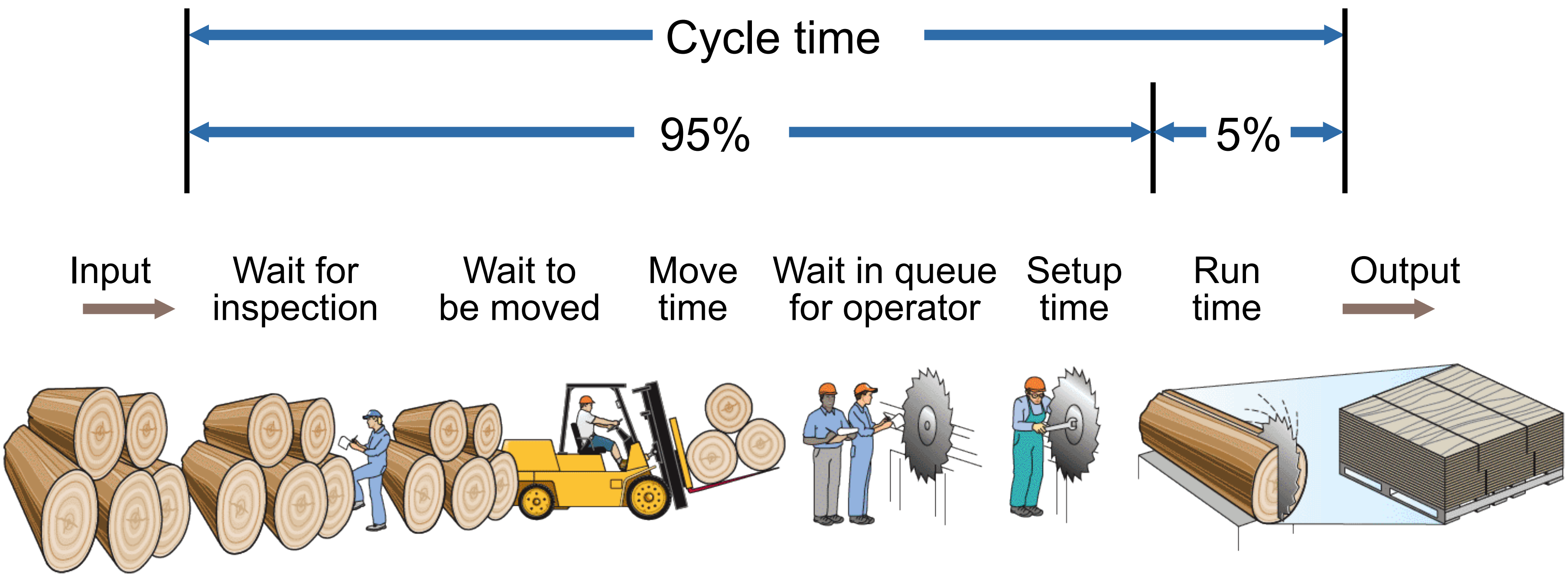
Why We Do Not Want to Hold Inventory

- Certain costs increase such as
  - Storage costs
  - insurance costs
  - outdate costs
  - large-lot quality cost
  - cost of production problems
- Ties capital for which the company pays interest
- Hides productivity and quality problems
- Risk of getting stuck with unsalable goods

Types of Inventory

- Raw material
  - Purchased but not processed
- Work-in-process (WIP)
  - Undergone some change but not completed
  - A function of cycle time for a product (e.g. items being transported)
- Maintenance/repair/operating (MRO)
  - Necessary to keep machinery and processes productive
- Finished goods
  - Completed product awaiting shipment

The Material Flow Cycle



Inventory and Service Quality

- Customers usually perceive quality service as *availability of goods* they want when they want them

- Inventory must be sufficient to provide high-quality customer service

Inventory Costs

Inventory-Related Costs

- Ordering costs (unit variable costs & fixed ordering costs)
  - costs of replenishing inventory, placing orders, receiving goods
  - costs for to prepare a machine or process for manufacturing an order
- Holding or Inventory carrying costs
  - cost of holding an item in inventory over time
- Shortage or Stock-out / penalty costs
  - How do you handle shortages?
  - Lost sales vs. backlogging
  - Watch out for service level
- Outdate costs (for perishable products)
- Opportunity costs

Holding Costs

	Determining Inventory Holding Costs
CATEGORY	**COST (AND RANGE) AS A PERCENT OF INVENTORY VALUE**
Housing costs (building rent or depreciation, operating costs, taxes, insurance)	6% (3 - 10%)
Material handling costs (equipment lease or depreciation, power, operating cost)	3% (1 - 3.5%)
Labor cost (receiving, warehousing, security)	3% (3 - 5%)
Investment costs (borrowing costs, taxes, and insurance on inventory)	11% (6 - 24%)
Pilferage, space, and obsolescence (much higher in industries undergoing rapid change like PCs and cell phones)	3% (2 - 5%)
Overall carrying cost	26%

Holding Costs

[redacted]	**Determining Inventory Holding Costs**
H… o…	
M… d…	
Lo…	3% (3 - 5%)
In… costs (borrowing costs, taxes, and insurance on inventory)	11% (6 - 24%)
Pilferage, space, and obsolescence (much higher in industries undergoing rapid change like PCs and cell phones)	3% (2 - 5%)
Overall carrying cost	26%

Holding costs vary considerably depending on the business, location, and interest rates. Generally greater than 15%, some high tech and fashion items have holding costs greater than 40%.

ABC Analysis

ABC Analysis

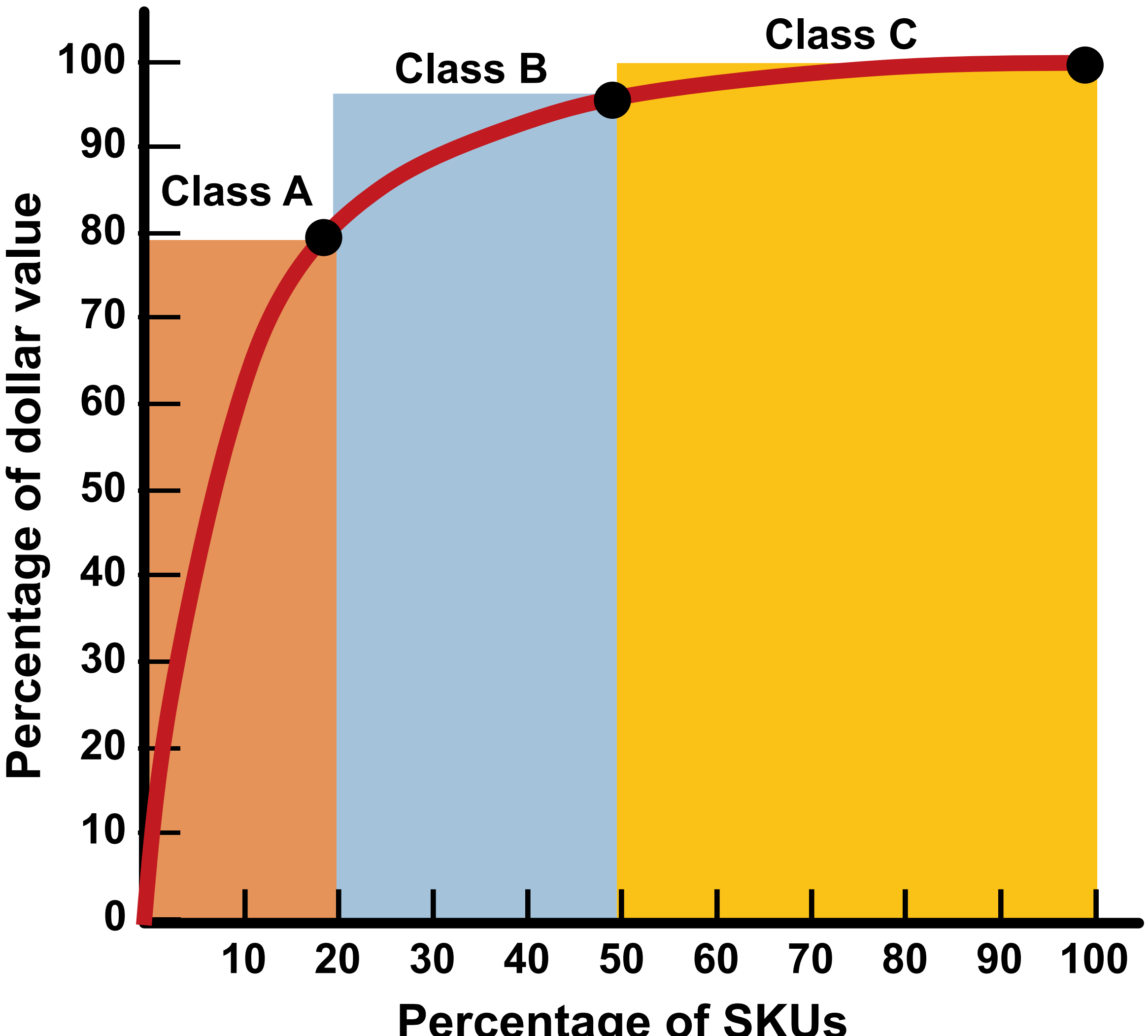
- Pay attention to your more critical products!
- Divides inventory into three classes based on annual dollar volume
  - Class A - high annual dollar volume
  - Class B - medium annual dollar volume
  - Class C - low annual dollar volume
- Used to establish policies that focus on the few critical parts and not the many trivial ones

ABC Analysis

- Concept: All items do not deserve the same attention in terms of inventory management
- Focus on items that have the highest monetary value

- Step 1. Start with the inventoried items ranked by dollar value in inventory in descending order
- Step 2. Plot the cumulative dollar/euro value in inventory versus the cumulative items in inventory

ABC Analysis



Typical Chart Using ABC Analysis

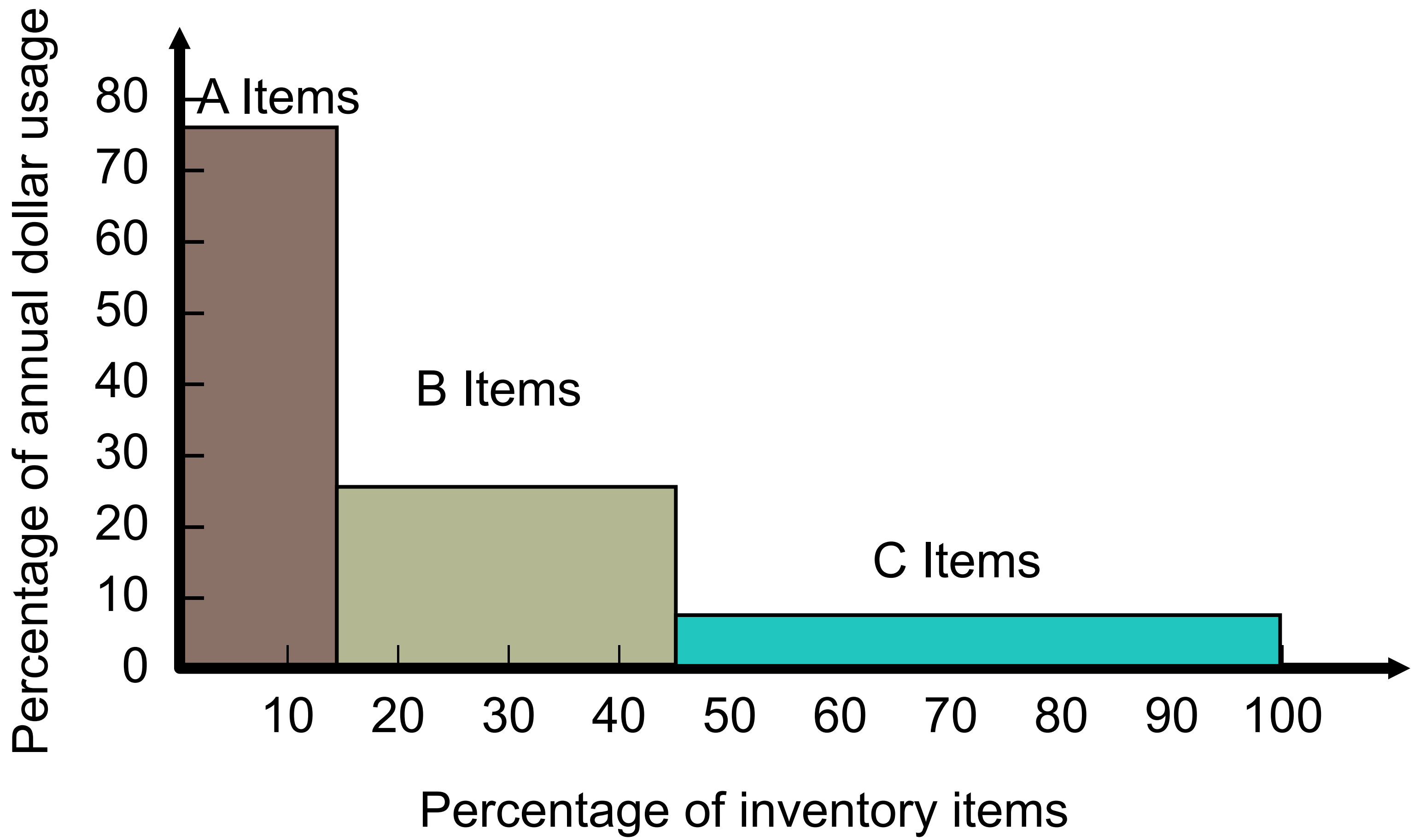
- Class A
  - 5 – 15 % of units
  - 70 – 80 % of value
- Class B
  - 30 % of units
  - 15 % of value
- Class C
  - 50 – 60 % of units
  - 5 – 10 % of value

ABC Analysis Example

ABC Calculation

(1) ITEM STOCK NUMBER	(2) PERCENT OF NUMBER OF ITEMS STOCKED	(3) ANNUAL VOLUME (UNITS)	x	(4) UNIT COST	=	(5) ANNUAL DOLLAR VOLUME	(6) PERCENT OF ANNUAL DOLLAR VOLUME		(7) CLASS
#10286	20%	1,000		$ 90.00		$ 90,000	38.8%	72%	A
#11526		500		154.00		77,000	33.2%		A
#12760		1,550		17.00		26,350	11.3%		B
#10867	30%	350		42.86		15,001	6.4%	23%	B
#10500		1,000		12.50		12,500	5.4%		B
#12572		600		$ 14.17		$ 8,502	3.7%		C
#14075		2,000		.60		1,200	.5%		C
#01036	50%	100		8.50		850	.4%	5%	C
#01307		1,200		.42		504	.2%		C
#10572		250		.60		150	.1%		C
		8,550				$232,057	100.0%		

ABC Analysis



ABC Analysis

- Other criteria than annual dollar volume may be used
  - High shortage or holding cost
  - Anticipated engineering changes
  - Delivery problems
  - Quality problems

ABC Analysis

- Policies employed may include
  1. More emphasis on supplier development for A items
  2. Tighter physical inventory control for A items
  3. More care in forecasting A items

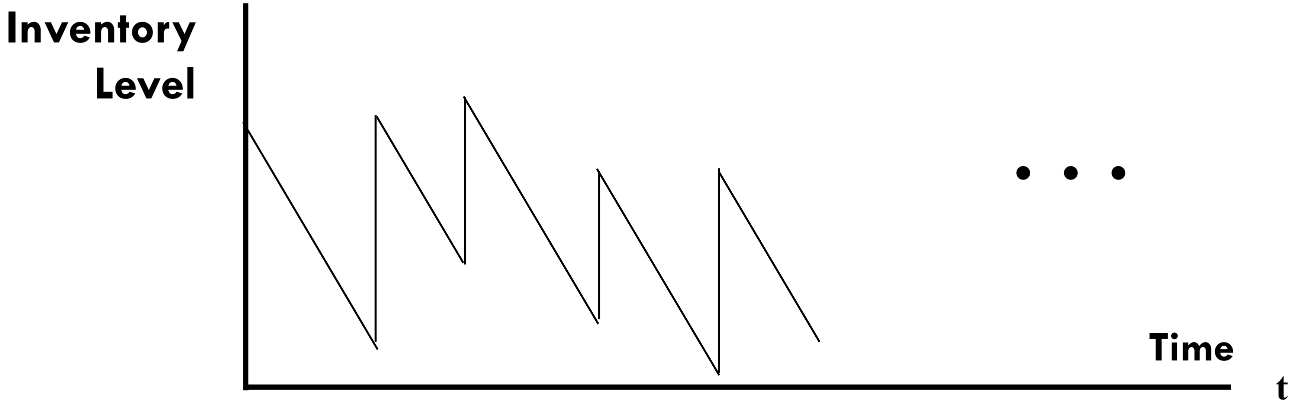
EOQ Models

Ordering Policy under constant demand

- Simple case
  1. Demand rate is constant and known with certainty
  2. Unit ordering cost = C
  3. Every time an order is placed, there is a fixed cost = S
  4. There is a unit holding cost = H
  5. No constraints are placed on the size of each order
  6. The lead time is zero

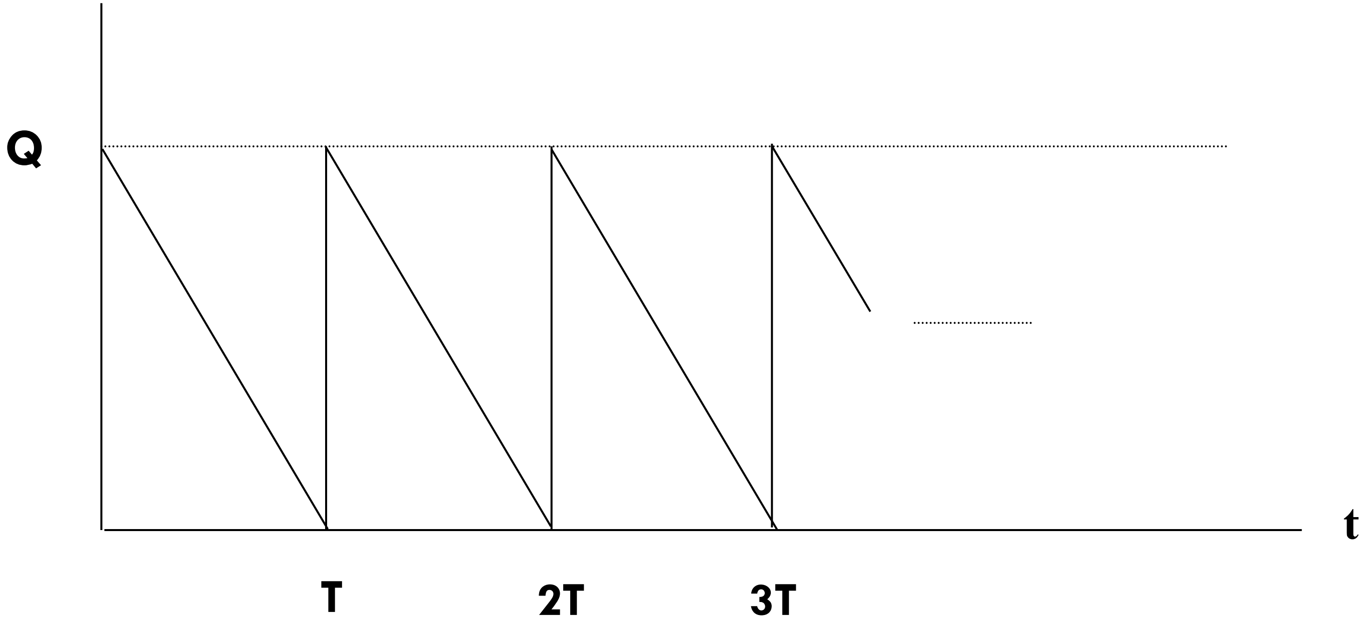
What is wrong with this management?

- Company with steady rate of demand D = 100 tons/month
- Total annual demand = 1200 tons
- Purchase price C = $250/ton
- Delivery costs S = $50 (each time)
- Holding costs (storage, insurance, …) H = $4/ton/month

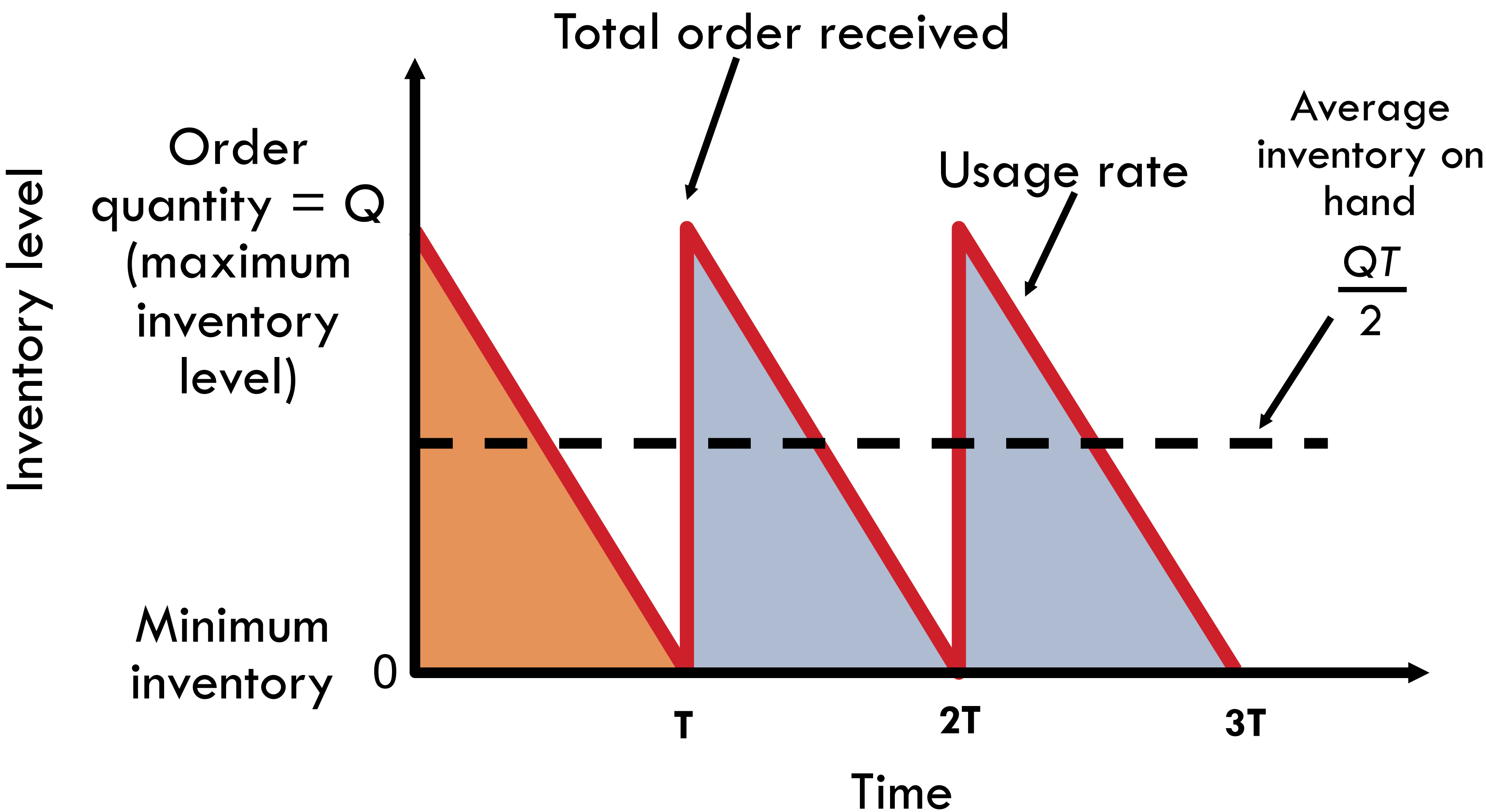


Get rid of pre-conceived ideas …

- Irrational Ordering (time, quantity) … why?
- Safety stock … why?

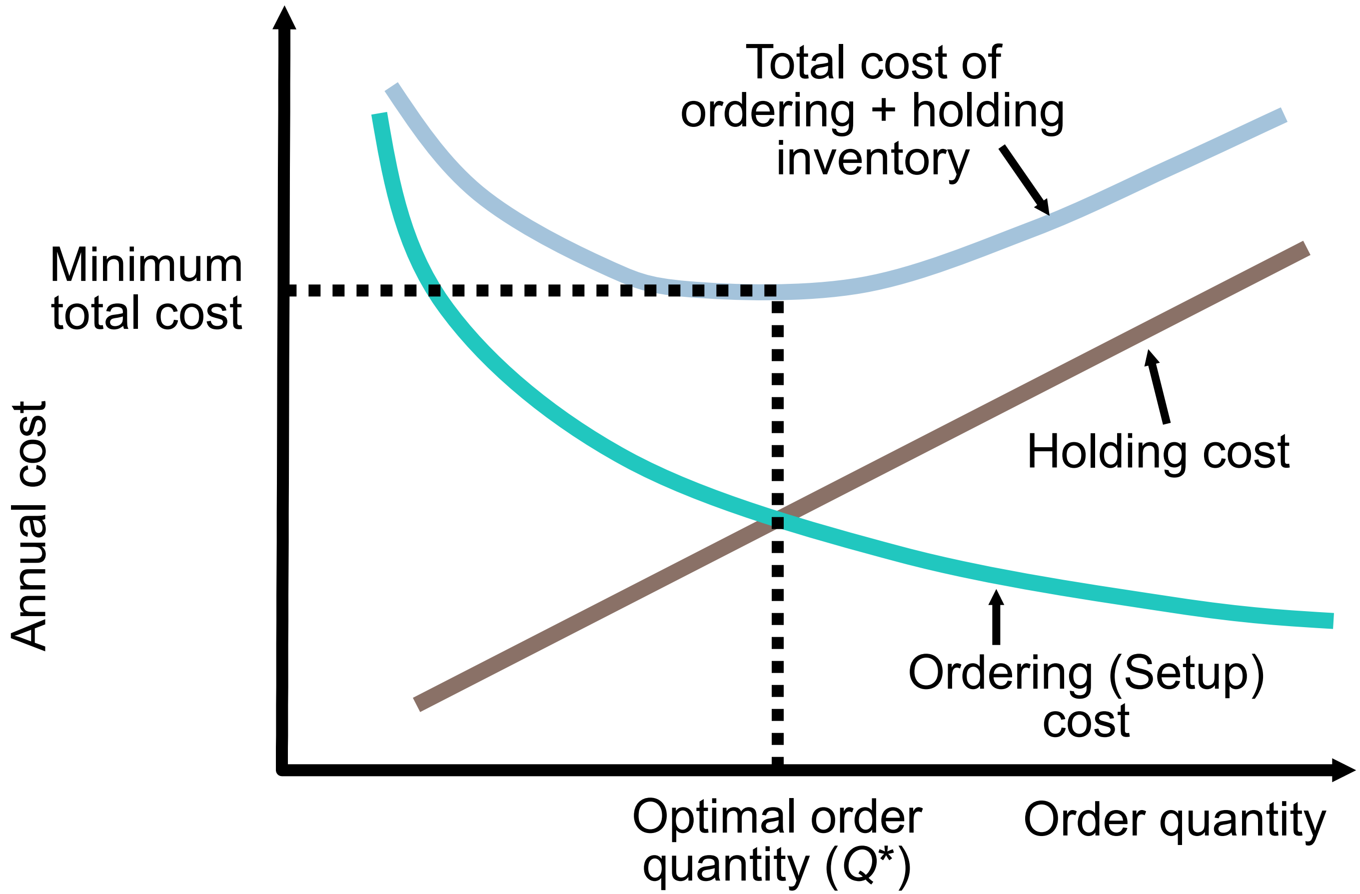


Inventory Usage Over Time



Determining the optimal cycle

Objective is to minimize Total Annual Cost



Minimizing Costs

- By minimizing the sum of setup (or ordering) and holding costs, total costs are minimized
- Optimal order size Q* will minimize total cost

- Optimal order quantity occurs when:
  - The derivative of the Total Cost with respect to the order quantity is equal to zero
  - The holding cost and setup cost are equal

Calculating the Annual Costs

- Annual holding cost

  **Annual holding cost = (Average cycle inventory)
  × (Unit holding cost)
  × No of orders placed / year**

- Annual ordering cost

  **Annual ordering cost = (Ordering cost / order)
  × No of orders placed / year**

- Total annual cycle-inventory cost

  **Total Annual costs = Annual holding cost
  + Annual ordering cost**

Calculating all the costs

Holding Cost / period

The cost of holding one unit in inventory for one cycle

$= H\,(QT)/2$

Ordering Cost / period

It is the cost of ordering one lot with Q units

$= CQ + S$

No. of orders / year

= Annual Demand / Oder Size = $12D/Q$

Total Cost (C)

It is the sum of annual holding and annual setup cost

Calculating the EOQ

- **Total annual cycle-inventory cost**

Fixed ordering cost

Variable ordering cost

Holding cost

$$TC = N\left[(S + CQ) + H\frac{QT}{2}\right]$$

Where

TC = total annual cost
C = unit ordering annual cycle-inventory cost
Q = lot size
H = holding cost per unit per period
D = demand per period
S = fixed ordering or setup costs per lot
T = re-order period

Calculating the EOQ

$$TC = N[(S + CQ) + H(QT/2)] =$$

$$= (12DS/Q) + (12D/Q)CQ + (12D/Q)(HQ^2/2D) =$$

$$= 12DS/Q + 12DC + 6HQ$$

To find the optimal Quantity Q: Set derivative w.r.t Q = 0

Therefore,

$$-(12DS/Q^2) + 6H = 0$$

The optimal - order- quantity

$$Q^* = \sqrt{2SD/H} = 50 \text{ tons}$$

T = 0,5 month

An EOQ Example

Determine the **optimal number of units to order**

D = 1,000 units per year

S = \$10 per order

H = \$.50 per unit per year

$$Q^* = \sqrt{\frac{2DS}{H}}$$

$$Q^* = \sqrt{\frac{2(1{,}000)(10)}{0.50}} = \sqrt{40{,}000} = 200 \text{ units}$$

An EOQ Example

Determine **expected number of orders**

D = 1,000 units

S = \$10 per order

H = \$.50 per unit per year

Q^* = 200 units

$$\text{Expected number of orders} = N = \frac{\text{Demand}}{\text{Order quantity}} = \frac{D}{Q^*}$$

$$N = \frac{1{,}000}{200} = 5 \text{ orders per year}$$

An EOQ Example

Determine **optimal time between orders**

D = 1,000 units

S = \$10 per order

H = \$.50 per unit per year

Q^* = 200 units

N = 5 orders/year

$$\text{Expected time between orders} = T = \frac{\text{Number of working days per year}}{\text{Expected number of orders}}$$

$$T = \frac{250}{5} = 50 \text{ days between orders}$$

An EOQ Example

Determine the **total annual cost**

D = 1,000 units

S = \$10 per order

H = \$.50 per unit per year

Q^* = 200 units

N = 5 orders/year

T = 50 days

Total annual cost = Setup cost + Holding cost

$$TC = \frac{D}{Q}S + \frac{Q}{2}H$$

$$= \frac{1{,}000}{200}(\$10) + \frac{200}{2}(\$.50)$$

$$= (5)(\$10) + (100)(\$.50)$$

$$= \$50 + \$50 = \$100$$

Note: the cost of materials is not included, as it is assumed that the demand will be satisfied and therefore it is a fixed cost

Calculating EOQ

EXAMPLE 1

A museum of natural history opened a gift shop which operates 52 weeks per year. Managing inventories has become a problem. Top-selling SKU is a bird feeder. Sales are 18 units per week, the supplier charges $60 per unit. Ordering cost is $45. Annual holding cost is 25 percent of a feeder's value. Management chose a 390-unit lot size.

What is the annual cycle-inventory cost of the current policy of using a 390-unit lot size?

Would a lot size of 468 be better?

Calculating EOQ

SOLUTION

We begin by computing the annual demand and holding cost as

$$D = \text{(18 units/week)(52 weeks/year) = 936 units}$$

$$H = 0.25(\$60/\text{unit}) = \$15$$

The total annual cycle-inventory cost for the current policy is

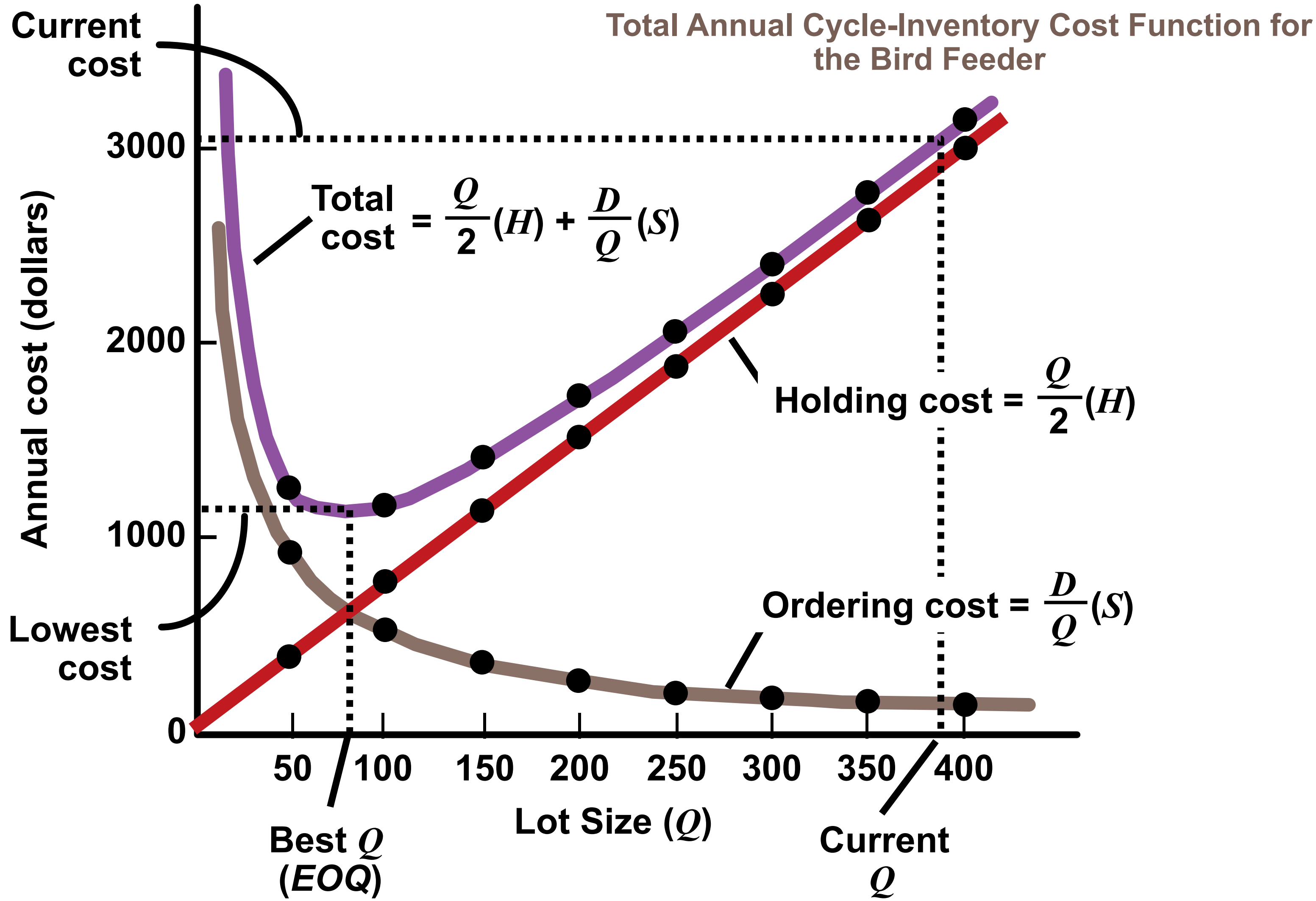
$$C = \frac{Q}{2}(H) + \frac{D}{Q}(S) = \frac{390}{2}(\$15) + \frac{936}{390}(\$45)$$

$$= \$2{,}925 + \$108 = \$3{,}033$$

The total annual cycle-inventory cost for the alternative lot size is

$$C = \frac{468}{2}(\$15) + \frac{936}{468}(\$45) = \$3{,}510 + \$90 = \$3{,}600$$

Calculating the EOQ



Finding the EOQ, Total Cost, TBO

EXAMPLE 2

For the bird feeders in Example 1, calculate the EOQ and its total annual cycle-inventory cost. How frequently will orders be placed if the EOQ is used?

SOLUTION

Using the formulas for EOQ and annual cost, we get

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(936)(45)}{15}} = 74.94 \quad \text{or} \quad 75 \text{ units}$$

Finding the EOQ, Total Cost, TBO

The total annual cost is much less than the $3,033 cost of the current policy of placing 390-unit orders.

Parameters

Current Lot Size (Q)	390	Economic Order Quantity	75
Demand (D)	936		
Order Cost (S)	$45		
Unit Holding Cost (H)	$15		

Annual Costs		**Annual Costs based on EOQ**	
Orders per Year	2.4	Orders per Year	12.48
Annual Ordering Cost	$108.00	Annual Ordering Cost	$561.60
Annual Holding Cost	$2,925.00	Annual Holding Cost	$562.50
Annual Inventory Cost	$3,033.00	Annual Inventory Cost	$1,124.10

Finding the EOQ, Total Cost, TBO

When the EOQ is used, the TBO can be expressed in various ways for the same time period.

$$TBO_{EOQ} = \frac{EOQ}{D} = \frac{75}{936} = 0.080 \text{ year}$$

$$TBO_{EOQ} = \frac{EOQ}{D}(12 \text{ months/year}) = \frac{75}{936}(12) = 0.96 \text{ month}$$

$$TBO_{EOQ} = \frac{EOQ}{D}(52 \text{ weeks/year}) = \frac{75}{936}(52) = 4.17 \text{ weeks}$$

$$TBO_{EOQ} = \frac{EOQ}{D}(365 \text{ days/year}) = \frac{75}{936}(365) = 29.25 \text{ days}$$

Finding the EOQ, Total Cost, TBO

EXAMPLE 3

Suppose that you are reviewing the inventory policies on an $80 item stocked at a hardware store. The current policy is to replenish inventory by ordering in lots of 360 units. Additional information is:

D = 60 units per week, or 3,120 units per year

S = $30 per order

H = 25% of selling price, or $20 per unit per year

What is the EOQ?

SOLUTION

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(3{,}120)(30)}{20}} = 97 \text{ units}$$

Finding the EOQ, Total Cost, TBO

What is the total annual cost of the current policy ($Q = 360$), and how does it compare with the cost with using the EOQ?

Current Policy	EOQ Policy
Q = 360 units	Q = 97 units
C = (360/2)(20) + (3,120/360)(30)	C = (97/2)(20) + (3,120/97)(30)
C = 3,600 + 260	C = 970 + 965
C = \$3,860	C = \$1,935

Finding the EOQ, Total Cost, TBO

What is the time between orders (TBO) for the current policy and the EOQ policy, expressed in weeks?

$$\textbf{TBO}_{360} = \frac{360}{3{,}120}\text{(52 weeks per year)} = 6 \text{ weeks}$$

$$\textbf{TBO}_{EOQ} = \frac{97}{3{,}120}\text{(52 weeks per year)} = 1.6 \text{ weeks}$$

Managerial Insights

SENSITIVITY ANALYSIS OF THE EOQ				
Parameter	**EOQ**	**Parameter Change**	**EOQ Change**	**Comments**
Demand	$\sqrt{\frac{2DS}{H}}$	↑	↑	**Increase in lot size is in proportion to the square root of *D*.**
Order/Setup Costs	$\sqrt{\frac{2DS}{H}}$	↓	↓	**Weeks of supply decreases and inventory turnover increases because the lot size decreases.**
Holding Costs	$\sqrt{\frac{2DS}{H}}$	↓	↑	**Larger lots are justified when holding costs decrease.**

Robustness

- The EOQ model is robust
- It works even if all parameters and assumptions are not met
- The total cost curve is relatively flat in the area of the EOQ

Introducing delivery lag

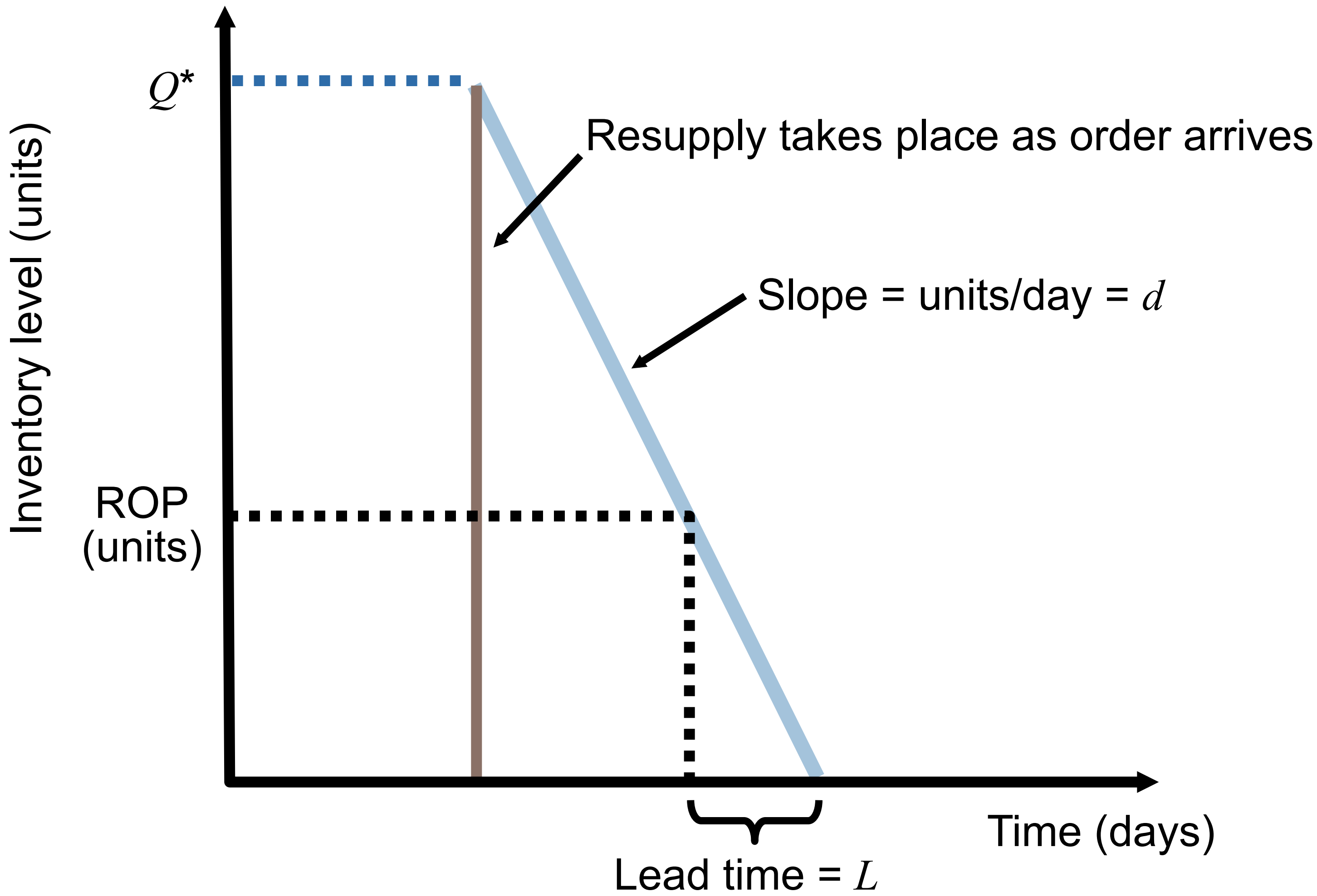
- EOQ answers the "how much" question
- The reorder point (ROP) tells "when" to order
- Lead time (L) is the time between placing and receiving an order

$$\text{ROP} = \begin{pmatrix}\text{Demand}\\ \text{per day}\end{pmatrix}\begin{pmatrix}\text{Lead time for a new}\\ \text{order in days}\end{pmatrix}$$

$$= d \times L$$

$$d = \frac{D}{\text{Number of working days in a year}}$$

Reorder Point Curve



Reorder Point Example

Demand = 8,000 iPods per year
250 working day year
Lead time for orders is 3 working days, but it may also take 4 days

$$d = \frac{D}{\text{Number of working days in a year}}$$

$$= 8{,}000/250 = 32 \text{ units}$$

$$\text{ROP} = d \times L$$

$$= 32 \text{ units per day} \times 3 \text{ days} = 96 \text{ units}$$

$$= 32 \text{ units per day} \times 4 \text{ days} = 128 \text{ units}$$

Introducing volume discounts

- A company buys re-writable DVDs (10 disks / box) from a large mail-order distributor
- The company uses approximately 5,000 boxes / year at a fairly constant rate
- The distributor offers the following quantity discount schedule:
  - If <500 boxes are ordered, then cost = \$10/box
  - If >500 but <800 boxes are ordered, then cost = \$9.50
  - If >800 boxes are ordered, then cost = \$9.25
- Fixed cost of purchasing = \$25, and the cost of capital = 12% per year. There is no storage cost.

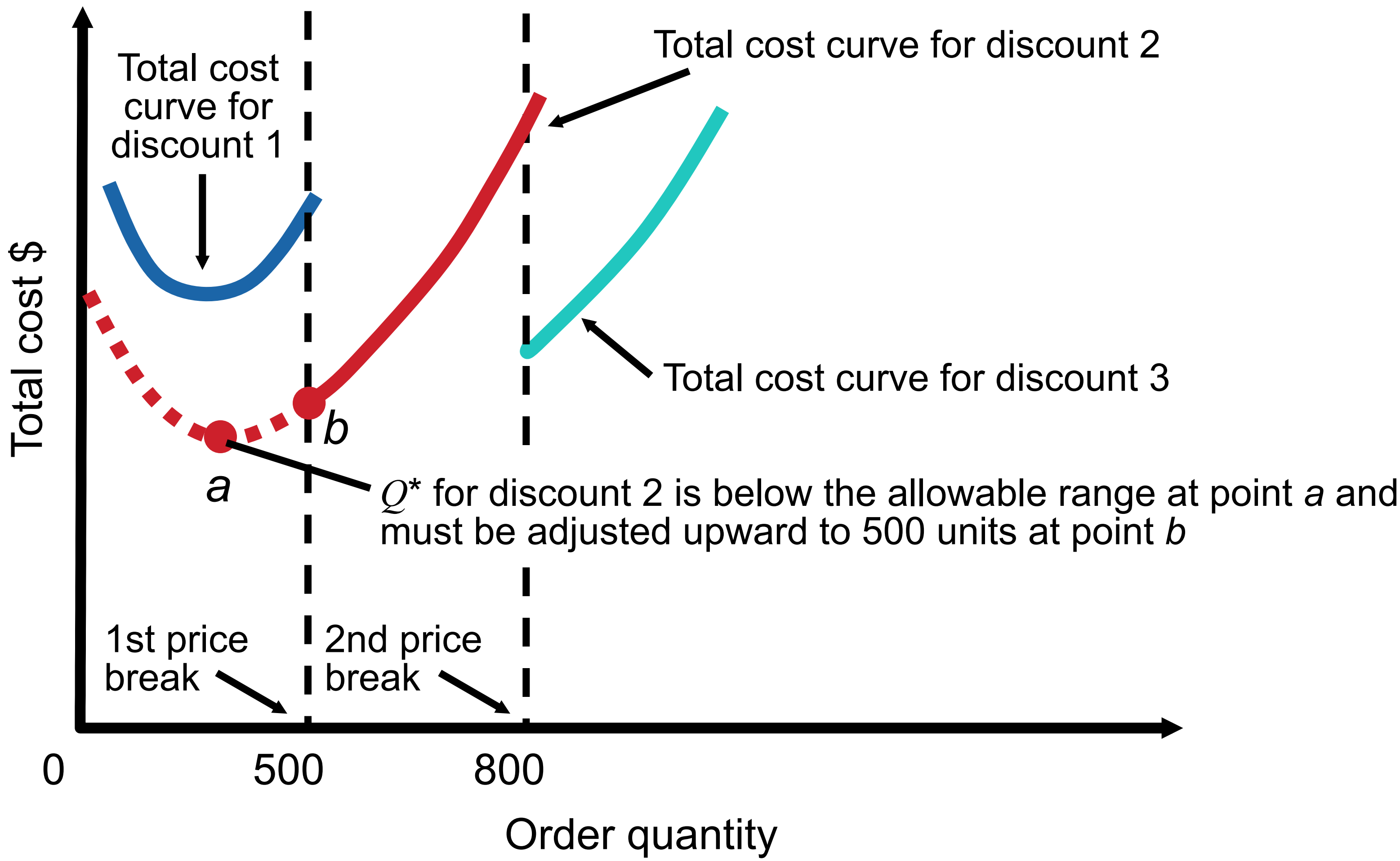
Introducing volume discounts

- Solve 3 EOQ models
- Each one will hold for the corresponding region; if it does not correspond, choose the lowest one that does
- Select the one with the lowest cost

Steps in analyzing a quantity discount

1. For each discount, calculate Q*
2. If Q* for a discount doesn't qualify, choose the lowest possible quantity to get the discount
3. Compute the total cost for each Q* or adjusted value from Step 2
4. Select the Q* that gives the lowest total cost

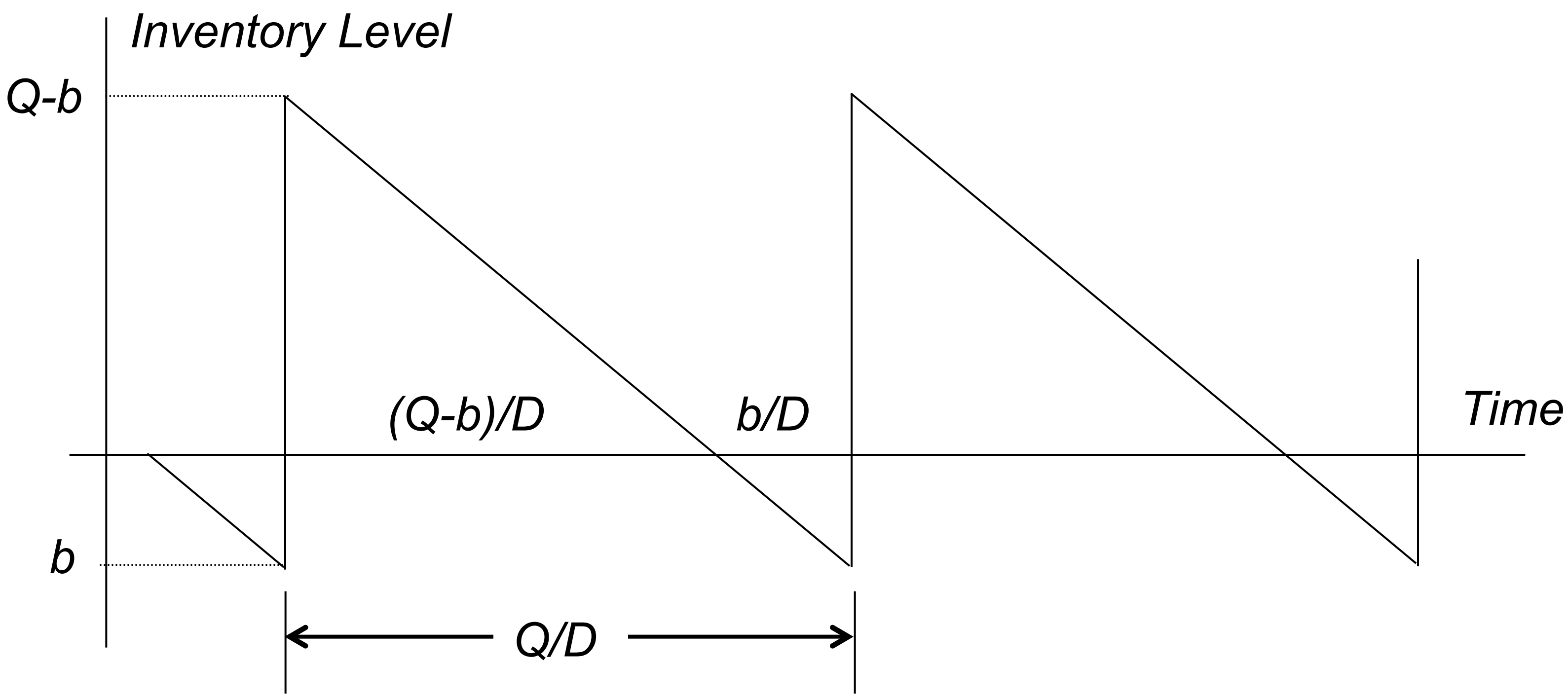
Quantity Discount Models



Allowing shortages

- A company is a mail-order distributor of audio CDs
- They sell about 50,000 CDs / year
- Each CD is packaged in a jewel box they buy from a supplier
- Fixed cost for an order of boxes = \$100; variable cost = \$0.50, storage cost = \$0.50/unit/year, and cost of money is 10%
- The company assumes that shortages are allowed, and lost demand is backlogged … it just gets to the customer a little later (!)
- The company assigns a "penalty" of \$1 for every week that a box is delivered late, so annual shortage cost (penalty) p = \$52/unit.

Allowing shortages



- Allow shortages up to b units
- Order quantity Q
- Order-up-to inventory = Q-b
- Reorder period = Q/D
- Period with I>0 = (Q-b)/D
- Period with I<0 = b/D

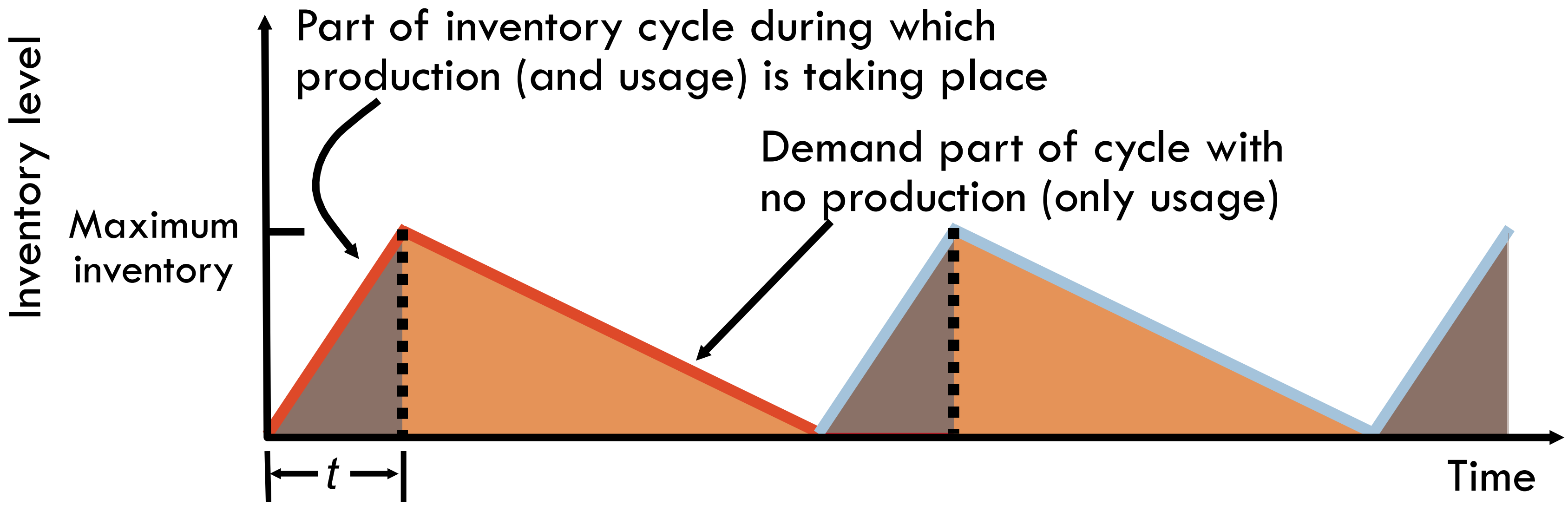
Allowing shortages

- Total Annual Cost = Ordering + Shortage + Holding costs
- Ordering cost = N [(S+CQ) + (pb(b/D)/2) + H(Q-b)((Q-b)/D)/2]
- Since N = D/Q, we have
- Total Annual Cost = $DS/Q + CD + (pb^2/2Q) + H(Q-b)^2/2D$

- To minimize, take derivative = 0, and solve
- $$hQ^2 - HbQ - (DS+pb^2) = 0$$

Production Order Quantity Model

1. Used when inventory builds up over a period of time after an order is placed
2. Used when units are produced and sold simultaneously



Production Order Quantity Model

Q = Number of pieces per order
H = Holding cost per unit per year
t = Length of the production run in days
p = Daily production rate
d = Daily demand/usage rate

$$\left(\begin{array}{c}\text{Annual inventory}\\ \text{holding cost}\end{array}\right) = (\text{Average inventory level}) \times \left(\begin{array}{c}\text{Holding cost}\\ \text{per unit per year}\end{array}\right)$$

$$\left(\begin{array}{c}\text{Average}\\ \text{inventory level}\end{array}\right) = (\text{Maximum inventory level})/2$$

$$\left(\begin{array}{c}\text{Maximum}\\ \text{inventory level}\end{array}\right) = \left(\begin{array}{c}\text{Total items produced}\\ \text{during the production}\\ \text{run}\end{array}\right) - \left(\begin{array}{c}\text{Total items used}\\ \text{during the}\\ \text{production run}\end{array}\right)$$

$$= pt - dt$$

Production Order Quantity Model

Q = Number of pieces per order

H = Holding cost per unit per year

t = Length of the production run in days

p = Daily production rate

d = Daily demand/usage rate

$$\begin{pmatrix}\text{Maximum}\\ \text{inventory level}\end{pmatrix} = \begin{pmatrix}\text{Total produced during}\\ \text{the production run}\end{pmatrix} - \begin{pmatrix}\text{Total used during}\\ \text{the production run}\end{pmatrix}$$

$$= \boldsymbol{pt - dt}$$

However, Q = total produced = pt ; thus $\boldsymbol{t = Q/p}$

$$\begin{pmatrix}\text{Maximum}\\ \text{inventory level}\end{pmatrix} = p\left(\frac{Q}{p}\right) - d\left(\frac{Q}{p}\right) = Q\left(1 - \frac{d}{p}\right)$$

$$\text{Holding cost} = \frac{\text{Maximum inventory level}}{2}(H) = \frac{Q}{2}\left[1 - \left(\frac{d}{p}\right)\right]H$$

Production Order Quantity Model

Q = Number of pieces per order
H = Holding cost per unit per year
t = Length of the production run in days
p = Daily production rate
d = Daily demand/usage rate

$$\text{Setup cost} = (D/Q)S$$

$$\text{Holding cost} = \tfrac{1}{2}HQ\left[1-\left(d/p\right)\right]$$

$$\frac{D}{Q}S = \tfrac{1}{2}HQ\left[1-\left(d/p\right)\right]$$

$$Q^2 = \frac{2DS}{H\left[1-\left(d/p\right)\right]}$$

$$Q_p^* = \sqrt{\frac{2DS}{H\left[1-\left(d/p\right)\right]}}$$

Remember, with no production taking place

$$Q^* = \sqrt{\frac{2DS}{H}}$$

Production Order Quantity Example

D =1,000 units
S =$10
H =$0.50 per unit per year

p = 8 units per day
d = 4 units per day

$$Q_p^* = \sqrt{\frac{2DS}{H\left[1-(d/p)\right]}}$$

$$Q_p^* = \sqrt{\frac{2(1{,}000)(10)}{0.50\left[1-(4/8)\right]}}$$

$$= \sqrt{\frac{20{,}000}{0.50(1/2)}} = \sqrt{80{,}000}$$

$$= 282.8 \text{ units, or } 283 \text{ units}$$

Production Order Quantity Model

Note:

$$d = 4 = \frac{D}{\text{Number of days the plant is in operation}} = \frac{1{,}000}{250}$$

When annual data are used the equation becomes

$$Q_p^* = \sqrt{\frac{2DS}{H\left(1 - \dfrac{\text{Annual demand rate}}{\text{Annual production rate}}\right)}}$$

Probabilistic Models and Safety Stock

Probabilistic Models and Safety Stock

- Demand is often UNCERTAIN
- The problem appears when there is LEAD TIME, L
- We have to set two parameters that define our ordering policy: Reorder Point (ROP) and Safety Stock (ss)
- You reorder when your inventory falls on or below ROP
- Use safety stock to achieve a desired service level and avoid stockouts

$$ROP = d \text{ x } L + ss$$

Expected Annual stockout costs = (expected units short/ cycle) x the stockout cost/unit x the number of orders per year

Safety Stock Example

Current policy:

ROP = 50 units

Orders per year = 6

Stockout cost = \$40 / unit

Carrying cost = \$5 / unit / year

Probability distribution for inventory demand during lead time

NUMBER OF UNITS (d X L)	PROBABILITY
30	.2
40	.2
Current ROP → 50	.3
60	.2
70	.1
	1.0

How much safety stock should we keep and added to 50 (current ROP)?

Safety Stock Example

ROP = 50 units
Orders /year = 6

Stockout cost = $40 /unit
Carrying cost = $5 /unit/ year

SAFETY STOCK	ADDITIONAL HOLDING COST	STOCKOUT COST	TOTAL COST
20	(20)($5) = $100	$0	$100
10	(10)($5) = $ 50	(10)(.1)($40)(6) = $240	$290
0	$ 0	(10)(.2)($40)(6) + (20)(.1)($40)(6) = $960	$960

A safety stock of 20 units gives the lowest total cost

ROP = 50 + 20 = 70 frames

Probabilistic Demand

Use prescribed service levels to set safety stock when the cost of stockouts cannot be determined

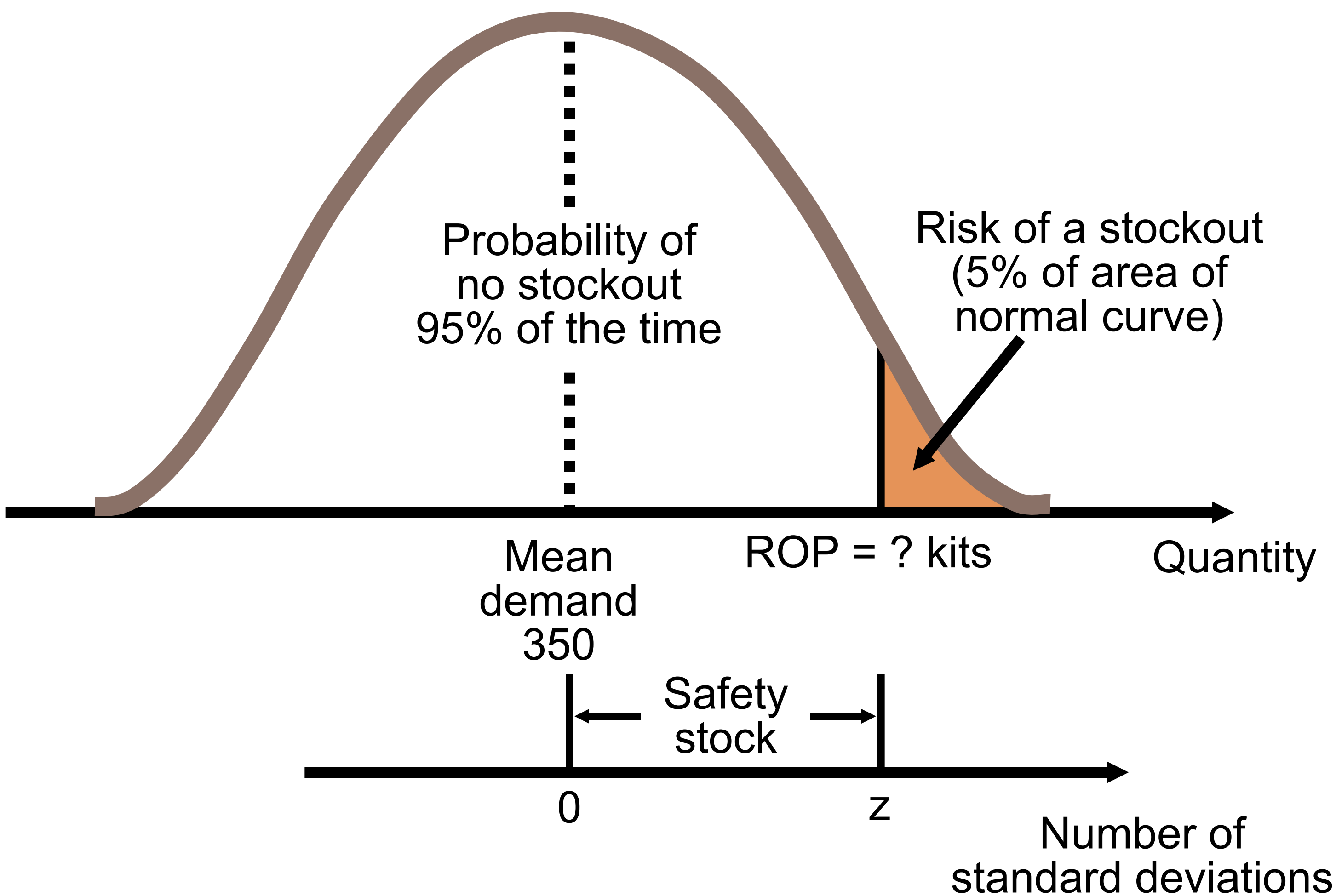
$$\text{ROP} = \text{demand during lead time} + Z\sigma_{dLT}$$

Where:

Z = Number of standard deviations

σ_{dLT} = Standard deviation of demand during lead time

Probabilistic Demand



Probabilistic Demand

μ = Average demand = 350 kits

σ_{dLT} = Standard deviation of demand during lead time = 10 kits

Z = 5% stockout policy (service level = 95%)

Using Normal distribution tables, for an area under the curve of 95%, the Z = 1.65

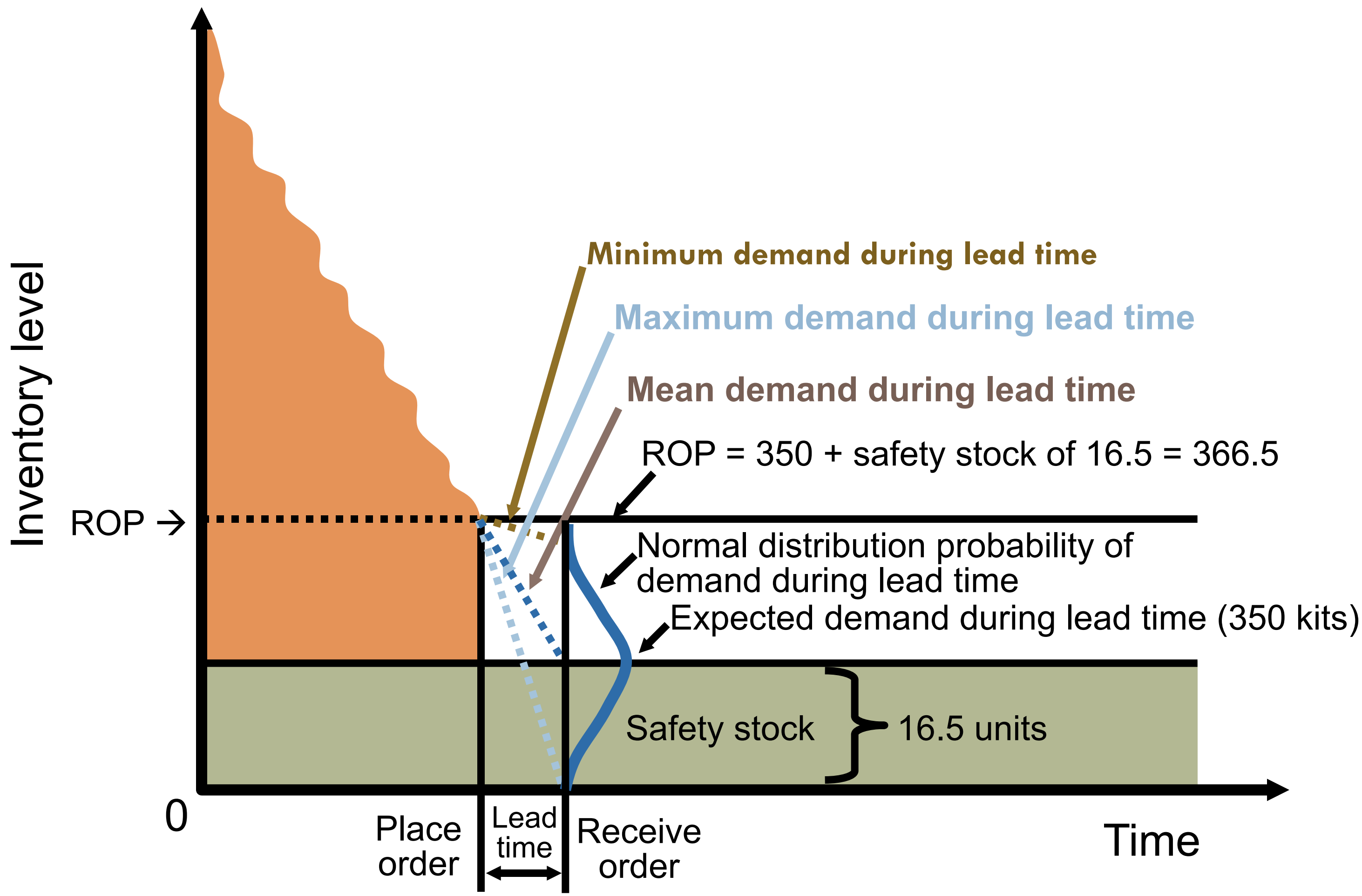
Safety stock = $Z\sigma_{dLT}$ = 1.65(10) = 16.5 kits

Reorder point = Expected demand during lead time + Safety stock

= 350 kits + 16.5 kits of safety stock

= 366.5 or 367 kits

Probabilistic Demand



Other Probabilistic Models

- When data on demand during lead time is not available, there are other models available
  1. When demand is variable and lead time is constant
  2. When lead time is variable and demand is constant
  3. When both demand and lead time are variable

Other Probabilistic Models: Variable demand, constant lead time

Demand is variable and lead time is constant

ROP = (*Average* daily demand x Lead time in days) + $Z\sigma_{dLT}$

where $\sigma_{dLT} = \sigma_d \sqrt{\text{Lead time}}$

σ_d = standard deviation of demand per day

Other Probabilistic Models: Variable demand, constant lead time

Average daily demand (normally distributed) = 15
Lead time in days (constant) = 2
Standard deviation of daily demand = 5
Service level = 90%

Z for 90% = 1.28
From Appendix I

$$\text{ROP} = (15 \text{ units x } 2 \text{ days}) + Z\sigma_{dLT}$$
$$= 30 + 1.28(5)(\sqrt{2})$$
$$= 30 + 9.02 = 39.02 \approx 39$$

Safety stock is about 9 computers

Other Probabilistic Models: Constant demand, variable lead time

$$\text{ROP} = (\text{Daily demand} \times \textit{Average} \text{ lead time in days}) + Z \times (\text{Daily demand}) \times \sigma_{LT}$$

where σ_{LT} = Standard deviation of lead time in days

Other Probabilistic Models: Constant demand, variable lead time

Daily demand (constant) = 10
Average lead time = 6 days
Standard deviation of lead time = σ_{LT} = 1
Service level = 98%, so Z (from Appendix I) = 2.055

$$\text{ROP} = (10 \text{ units} \times 6 \text{ days}) + 2.055(10 \text{ units})(1)$$
$$= 60 + 20.55 = 80.55$$

Reorder point is about 81 cameras

Other Probabilistic Models: Variable demand, variable lead time

ROP = (Average daily demand x Average lead time) + $Z\sigma_{dLT}$

where σ_d = Standard deviation of demand per day

σ_{LT} = Standard deviation of lead time in days

$$\sigma_{dLT} = \sqrt{(\text{Average lead time} \times \sigma_d^2) + (\text{Average daily demand})^2 \sigma^2_{LT}}$$

Other Probabilistic Models: Variable demand, variable lead time

Average daily demand (normally distributed) = 150
Standard deviation = σ_d = 16
Average lead time 5 days (normally distributed)
Standard deviation = σ_{LT} = 1 day
Service level = 95%, so Z = 1.65 (from Normal tables)

$$\text{ROP} = (150 \text{ packs} \times 5 \text{ days}) + 1.65\sigma_{dLT}$$

$$\sigma_{dLT} = \sqrt{\left(5 \text{ days} \times 16^2\right) + \left(150^2 \times 1^2\right)} = \sqrt{\left(5 \times 256\right) + \left(22{,}500 \times 1\right)}$$

$$= \sqrt{\left(1{,}280\right) + \left(22{,}500\right)} = \sqrt{23{,}780} \cong 154$$

$$\text{ROP} = (150 \times 5) + 1.65(154) \cong 750 + 254 = 1{,}004 \text{ packs}$$

Inventory Control Systems

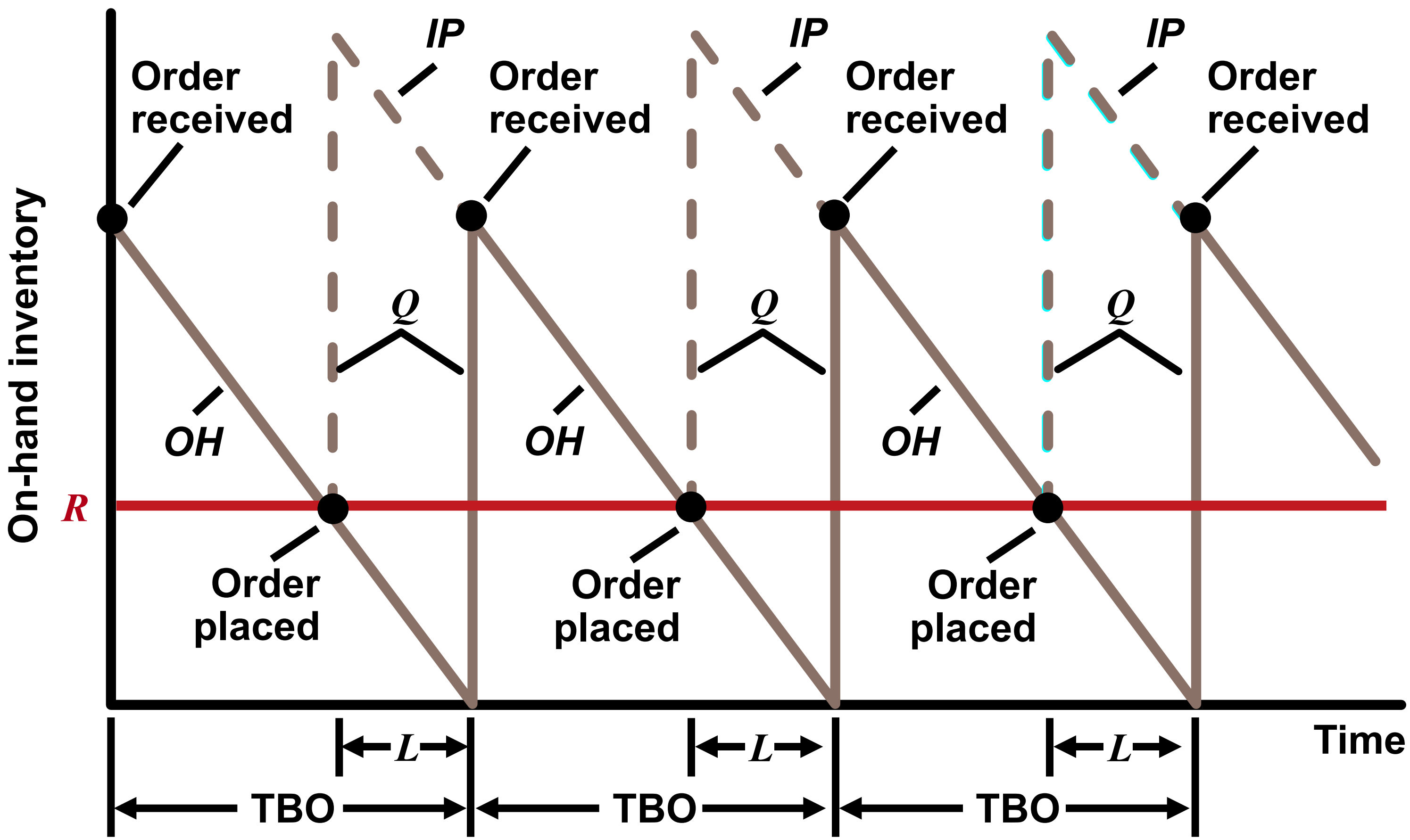
Inventory Control Systems

- Continuous review (Q) system
  - Reorder point system (ROP) and fixed order quantity system
  - For independent demand items (i.i.d.)
  - Tracks inventory position (IP)
  - Includes scheduled receipts (SR), on-hand inventory (OH), and back orders (BO)

Inventory position = On-hand inventory + Scheduled receipts – Backorders

$$IP = OH + SR - BO$$

Selecting the Reorder Point



Q System When Demand and Lead Time Are Constant and Certain

Continuous Review Systems

The on-hand inventory is only 10 units, and the reorder point R is 100. There are no backorders, but there is one open order for 200 units. Should a new order be placed?

SOLUTION

$$IP = OH + SR - BO = 10 + 200 - 0 = 210$$

$$R = 100$$

Decision: Do not place a new order

Continuous Review Systems

Reorder Point Level:

Assuming that the demand rate per period and the lead time are constant, the level of inventory at which a new order is placed (reorder point) can be calculated as follows:

$$R = dL$$

Where

d = demand rate per period
L = lead time

Remember: The order quantity Q is the EOQ!

Continuous Review Systems

EXAMPLE 4

Demand for chicken soup at a supermarket is always 25 cases a day and the lead time is always 4 days. The shelves were just restocked with chicken soup, leaving an on-hand inventory of only 10 cases. No backorders currently exist, but there is one open order in the pipeline for 200 cases. What is the inventory position? Should a new order be placed?

SOLUTION

R = **Total demand during lead time = (25)(4) = 100 cases**

$$IP = OH + SR - BO$$

$$= 10 + 200 - 0 = 210 \text{ cases}$$

Decision: Do not place a new order

Continuous Review Systems

- Selecting the reorder point with *variable demand* and constant lead time

Reorder point = Average demand during lead time + Safety stock

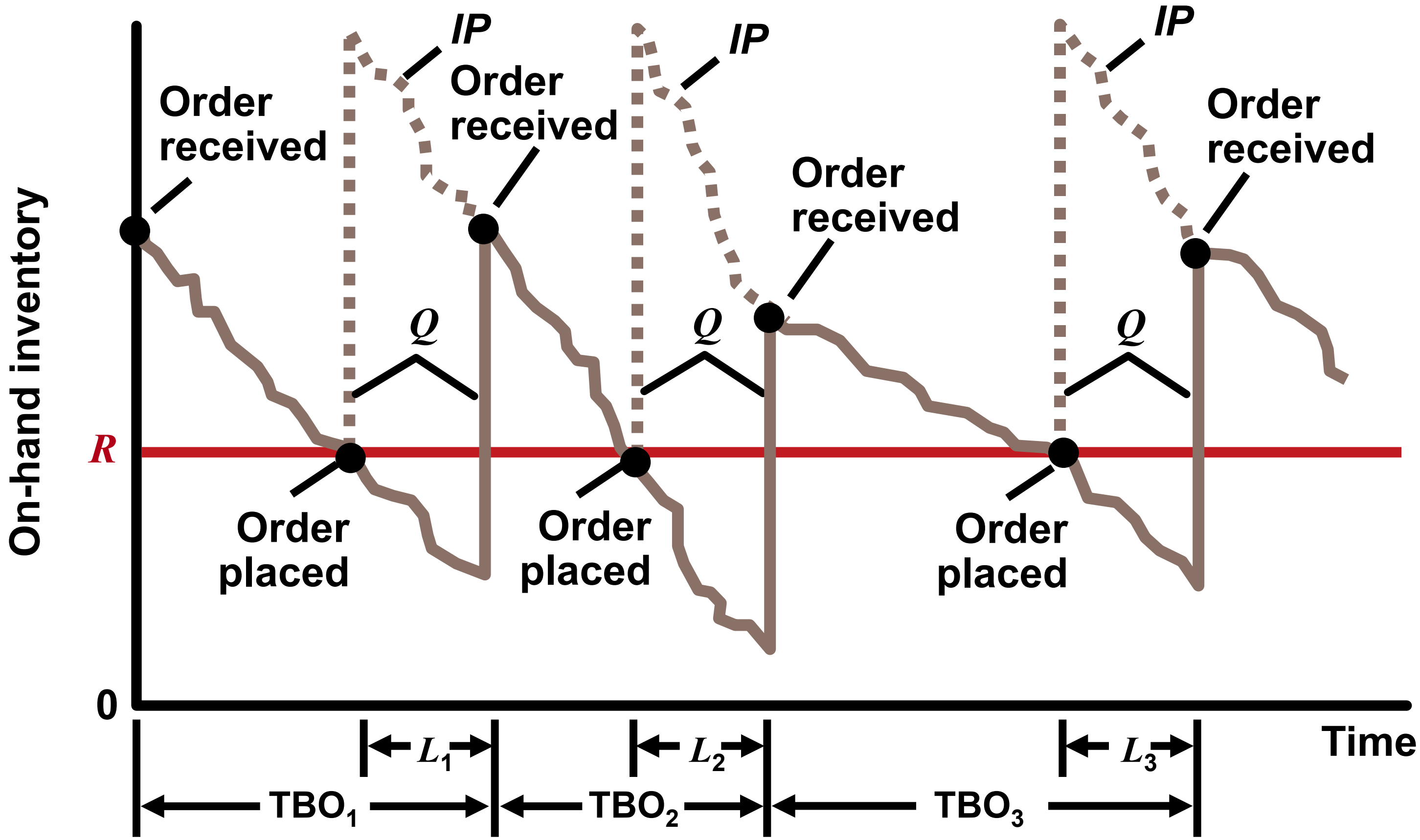
$$= \bar{d}L + \text{safety stock}$$

where

$\bar{d}$ = **average demand per week (or day or months)**

L = **constant lead time in weeks (or days or months)**

Continuous Review Systems (uncertain demand)



Q System When Demand Is Uncertain

How to determine the Reorder Point

1. Choose an appropriate service-level policy
   - Select service level or cycle service level
   - Protection interval
2. Determine the demand during lead time probability distribution
3. Determine the safety stock and reorder point levels

Demand During Lead Time

- Specify mean $\bar{d}$ and standard deviation σ_d for the demand (typically these values are given)
- Calculate standard deviation of demand during lead time L

$$\sigma_{dLT} = \sqrt{\sigma_d^2 L} = \sigma_d \sqrt{L}$$

- Then, the safety stock and reorder point are

$$\boxed{\textbf{Safety stock} = z\sigma_{dLT}}$$

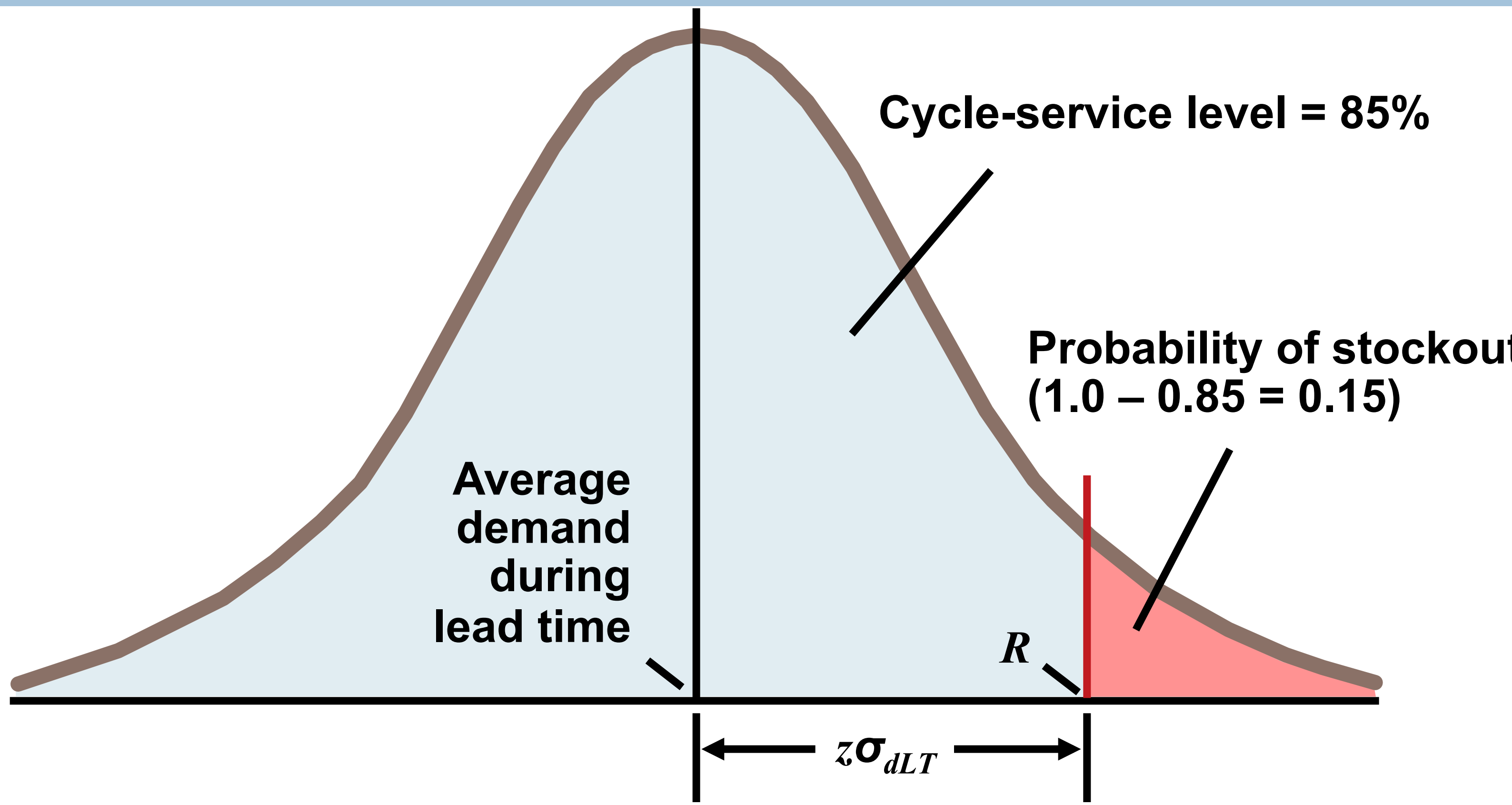
where

z = **number of standard deviations needed to achieve the cycle-service level (found from tables)**

σ_{dLT} = **stand deviation of demand during lead time**

$$\textbf{Reorder point} = R = \bar{d}L + \textbf{safety stock}$$

Demand During Lead Time



Finding Safety Stock with a Normal Probability Distribution for an 85 Percent Cycle-Service Level

Reorder Point for Variable Demand

EXAMPLE 5

Let us return to the bird feeder in Example 2.

The EOQ is 75 units.

Suppose that the average demand is 18 units per week with a standard deviation of 5 units.

The lead time is constant at two weeks.

Determine the safety stock and reorder point if management wants a 90 percent cycle-service level.

Reorder Point for Variable Demand

SOLUTION

In this case, σ_d = 5, $\bar{d}$ = 18 units, and L = 2 weeks, so $\sigma_{dLT} = \sigma_d\sqrt{L} = 5\sqrt{2} = 7.07$. Consult the body of the table in the Normal Distribution appendix for 0.9000, which corresponds to a 90 percent cycle-service level. The closest number is 0.8997, which corresponds to 1.2 in the row heading and 0.08 in the column heading. Adding these values gives a z value of 1.28. With this information, we calculate the safety stock and reorder point as follows:

Safety stock = $z\sigma_{dLT}$ = 1.28(7.07) = 9.05 or 9 units

Reorder point = $\bar{d}L$ + Safety stock = 2(18) + 9 = 45 units

Reorder Point for Variable Demand

EXAMPLE 6

Suppose that the demand during lead time is normally distributed with an average of 85 and $\sigma_{dLT} = 40$. Find the safety stock, and reorder point R, for a 95 and 85 percent cycle-service level.

SOLUTION

Safety stock = $z\sigma_{dLT}$ = 1.645(40) = 65.8 or 66 units

R = Average demand during lead time + Safety stock

R = 85 + 66 = 151 units

Find the safety stock, and reorder point R, for an 85 percent cycle-service level.

Safety stock = $z\sigma_{dLT}$ = 1.04(40) = 41.6 or 42 units

R = Average demand during lead time + Safety stock

R = 85 + 42 = 127 units

Reorder Point for Variable Demand & Variable Lead Time

- Often the case that both are variable
- The equations are more complicated

$$\text{Safety stock} = z\sigma_{dLT}$$

$$R = (\text{Average weekly demand} \times \text{Average lead time}) + \text{Safety stock}$$

$$= \bar{d}\bar{L} + \text{Safety stock}$$

where

$\bar{d}$ = Average weekly (or daily or monthly) demand

$\bar{L}$ = Average lead time

σ_d = Standard deviation of weekly (or daily or monthly) demand

σ_{LT} = Standard deviation of the lead time

$$\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2}$$

Reorder Point for Variable Demand & Variable Lead Time

EXAMPLE 7

The Office Supply Shop estimates that the average demand for a popular ball-point pen is 12,000 pens per week with a standard deviation of 3,000 pens. The current inventory policy calls for replenishment orders of 156,000 pens. The average lead time from the distributor is 5 weeks, with a standard deviation of 2 weeks. If management wants a 95 percent cycle-service level, what should the reorder point be?

Reorder Point for Variable Demand & Variable Lead Time

SOLUTION

We have $\bar{d}$ = 12,000 pens, σ_d = 3,000 pens, $\bar{L}$ = 5 weeks, and σ_{LT} = 2 weeks

$$\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2} = \sqrt{(5)(3{,}000)^2 + (12{,}000)^2(2)^2}$$

$$= 24{,}919.87 \text{ pens}$$

From the Normal Distribution appendix for 0.9500, the appropriate z value = 1.65. We calculate the safety stock and reorder point as follows:

$$\text{Safety stock} = z\sigma_{dLT} = (1.65)(24{,}919.87)$$
$$= 41{,}117.79 \text{ or } 41{,}118 \text{ pens}$$

$$\text{Reorder point} = \bar{d}\bar{L} + \text{Safety stock} = (12{,}000)(5) + 41.118$$
$$= 101{,}118 \text{ pens}$$

Reorder Point for Variable Demand & Variable Lead Time

EXAMPLE 8

Grey Wolf lodge is a popular 500-room hotel in the North Woods. Managers need to keep close tabs on all of the room service items, including a special pint-scented bar soap. The daily demand for the soap is 275 bars, with a standard deviation of 30 bars. Ordering cost is $10 and the inventory holding cost is $0.30/bar/year. The lead time from the supplier is 5 days, with a standard deviation of 1 day. The lodge is open 365 days a year.

What should the reorder point be for the bar of soap if management wants to have a 99 percent cycle-service?

Reorder Point for Variable Demand & Variable Lead Time

SOLUTION

$$\bar{d} = 275 \text{ bars}$$

$$\bar{L} = 5 \text{ days}$$

$$\sigma_d = 30 \text{ bars}$$

$$\sigma_{LT} = 1 \text{ day}$$

$$\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2} = 283.06 \text{ bars}$$

From the Normal Distribution appendix for 0.9900, z = 2.33.
We calculate the safety stock and reorder point as follows;

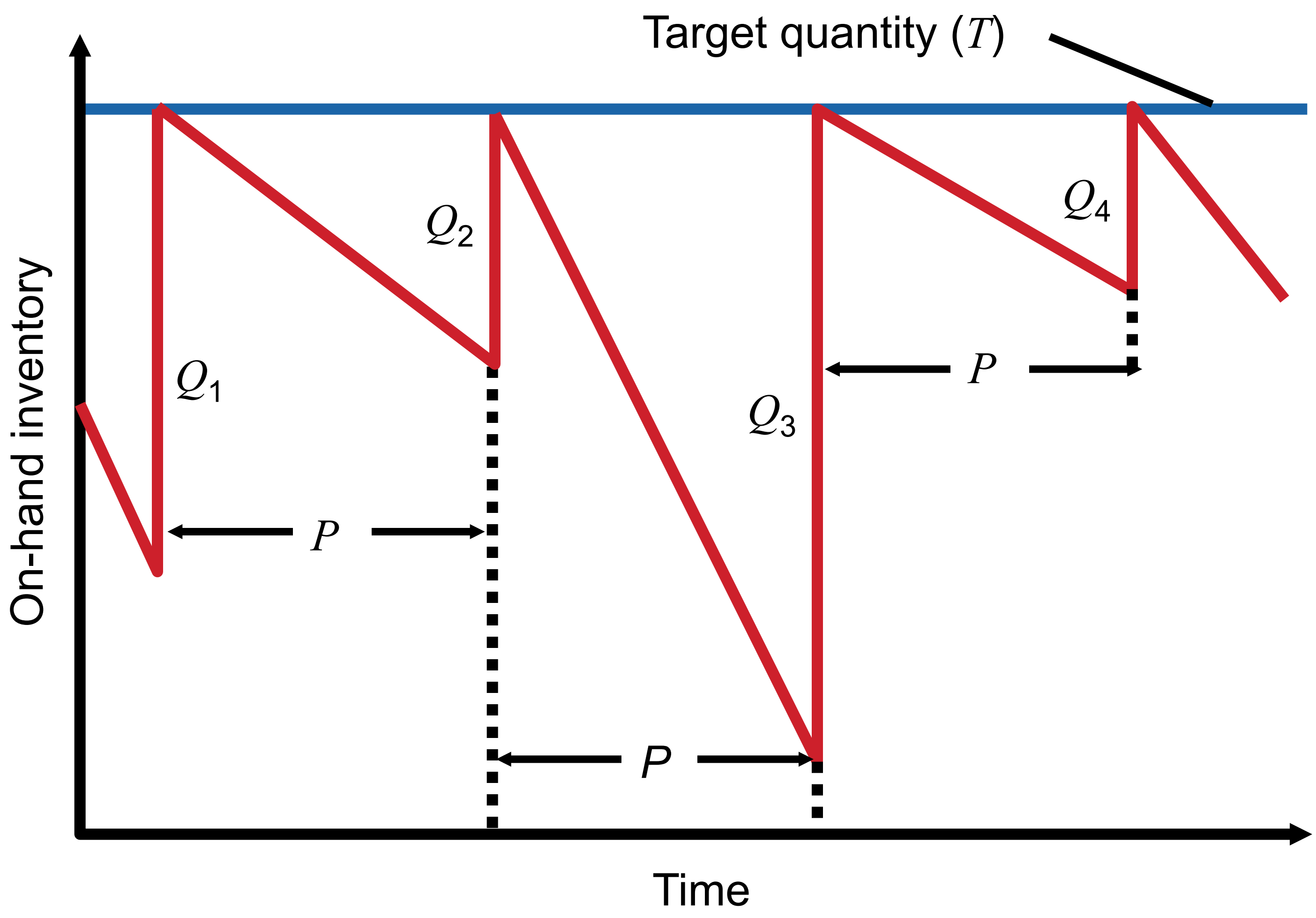
Safety stock = $z\sigma_{dLT}$ = (2.33)(283.06) = 659.53 or 660 bars

Reorder point + safety stock = $\bar{d}\bar{L}$ + safety stock
= (275)(5) + 660 = 2,035 bars

Periodic Review (or fixed period) System (P)

- Fixed interval reorder system or periodic reorder system
  - Four of the original EOQ assumptions maintained
    - No constraints are placed on lot size
    - Holding and ordering costs
    - Independent demand
    - Lead times are certain
- Order is placed to bring the inventory position up to the target inventory level, *T*, when the predetermined time, *P*, has elapsed
  - Only relevant costs are ordering and holding
  - Lead times are known and constant
  - Items are independent of one another

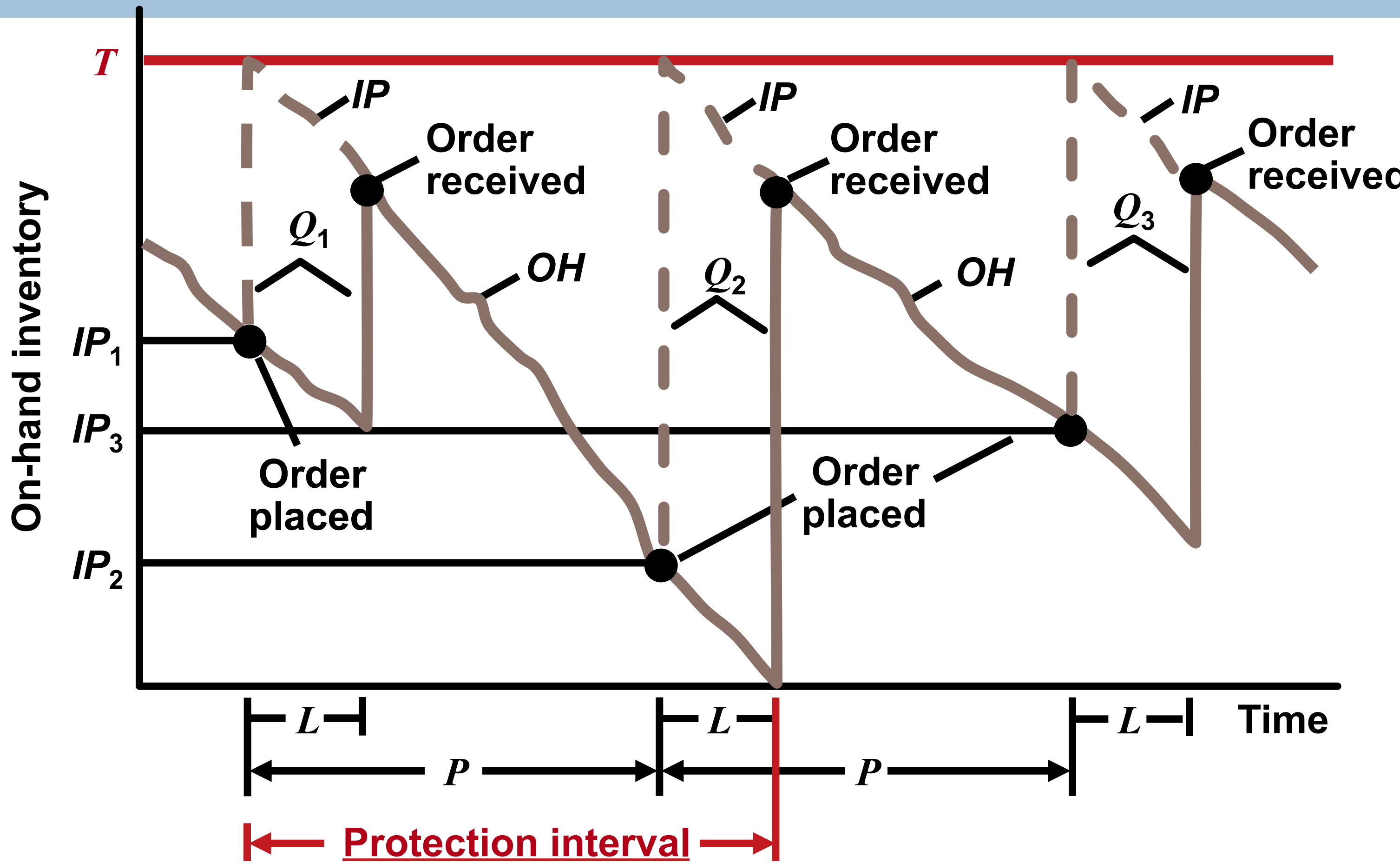
Periodic Review Systems



Periodic Review Systems

- Inventory is only counted at each review period
- May be scheduled at convenient times
- Appropriate in routine situations
- May result in stockouts between periods
- May require increased safety stock

Periodic Review System (P)



P System When Demand Is Uncertain

How Much to Order in a P System

EXAMPLE 9

A distribution center has a backorder for five 36-inch color TV sets. No inventory is currently on hand, and now is the time to review. How many should be reordered if $T = 400$ and no receipts are scheduled?

SOLUTION

$$IP = OH + SR - BO$$

$$= 0 + 0 - 5 = -5 \text{ sets}$$

$$T - IP = 400 - (-5) = 405 \text{ sets}$$

That is, 405 sets must be ordered to bring the inventory position up to T sets.

How Much to Order in a P System

EXAMPLE 10

The on-hand inventory is 10 units, and T is 400. There are no back orders, but one scheduled receipt of 200 units. Now is the time to review. How much should be reordered?

SOLUTION

$$IP = OH + SR - BO$$

$$= 10 + 200 - 0 = 210$$

$$T - IP = 400 - 210 = 190$$

The decision is to order 190 units

Periodic Review System

- Selecting the time between reviews, choosing P and T
- Selecting T when demand is variable and lead time is constant

IP covers demand over a protection interval of $P + L$

The average demand during the protection interval is $\bar{d}(P + L)$, or

$$T = \bar{d}(P + L) + \text{safety stock for protection interval}$$

Safety stock = $z\sigma_{P+L}$, *where* σ_{P+L} = $\sigma_d\sqrt{P+L}$

Calculating P and T

EXAMPLE 11

Again, let us return to the bird feeder example. Recall that demand for the bird feeder is normally distributed with a mean of 18 units per week and a standard deviation in weekly demand of 5 units. The lead time is 2 weeks, and the business operates 52 weeks per year. The Q system developed in Example 5 called for an EOQ of 75 units and a safety stock of 9 units for a cycle-service level of 90 percent. What is the equivalent P system? Answers are to be rounded to the nearest integer.

Calculating P and T

SOLUTION

We first define D and then P. Here, P is the time between reviews, expressed in weeks because the data are expressed as demand per week:

$$D = (18 \text{ units/week})(52 \text{ weeks/year}) = 936 \text{ units}$$

$$P = \frac{\text{EOQ}}{D}(52) = \frac{75}{936}(52) = 4.2 \text{ or } 4 \text{ weeks}$$

With $\bar{d}$ = 18 units per week, an alternative approach is to calculate P by dividing the EOQ by $\bar{d}$ to get 75/18 = 4.2 or 4 weeks. Either way, we would review the bird feeder inventory every 4 weeks.

Calculating P and T

We now find the standard deviation of demand over the protection interval $(P + L) = 6$:

$$\sigma_{P+L} = \sigma_d \sqrt{P+L} = 5\sqrt{6} = 12.25 \text{ units}$$

Before calculating T, we also need a z value. For a 90 percent cycle-service level z = 1.28. The safety stock becomes

Safety stock = $z\sigma_{P+L}$ = 1.28(12.25) = 15.68 or 16 units

We now solve for T:

T = Average demand during the protection interval + Safety stock

= $\bar{d}(P + L)$ + safety stock

= (18 units/week)(6 weeks) + 16 units = 124 units

Comparative Advantages

- Primary advantages of P systems
  - Convenient
  - Orders can be combined
  - Only need to know IP when review is made
- Primary advantages of Q systems
  - Review frequency may be individualized
  - Fixed lot sizes can result in quantity discounts
  - Lower safety stocks

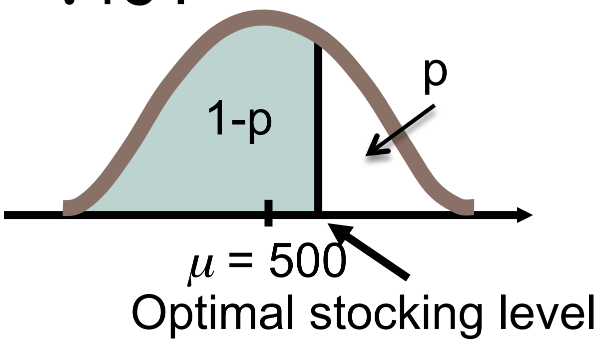
Single Period Model

Single-Period Model

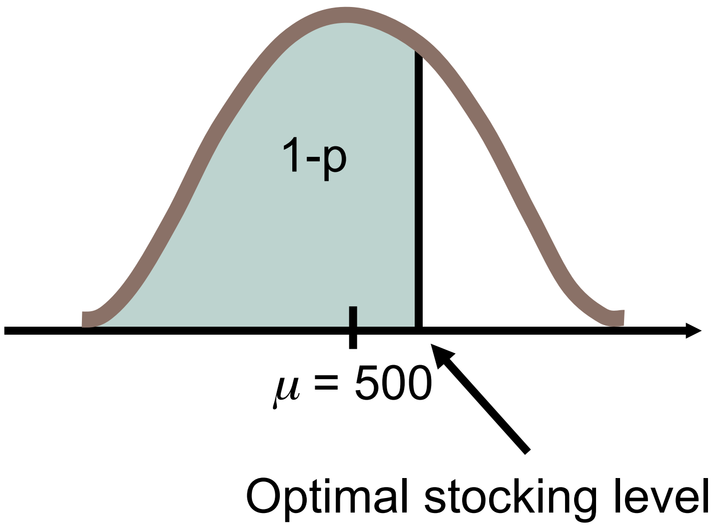
- Only one order is placed for a product
- Units have little or no value at the end of the sales period
- Newsboy problem 1:
- Demand = 500 papers/day, σ = 100 paper
- Cost to newsboy = 10c, Sales price = 30c
- How many papers should he order?

The newsboy problem

- If he sells a paper, he makes a profit = 20c
- If he doesn't sell a paper he makes a loss = 10c
- If he orders x papers, on the i-th paper he makes an expected profit:
- $E(\text{Profit})_i = 20p - 10(1\text{-}p)$, where p=sale of i-th paper
- Breakeven occurs when the Expected profit = 0
- So, $20p - 10(1\text{-}p) = 0$, and therefore $p^* = 1/3$
- By looking at the Normal tables, Z = .431



The newsboy problem



- Therefore:
- $Z = .431 = (X^* - \mu)/\sigma = (X^*-500)/100$
  - $X^* = 543$

A small variation

- Assume that he can return the paper, if unsold for 5c each
- $E(\text{Profit})_i = 20p - 5(1\text{-}p)$, where p=sale of i-th paper
- Breakeven occurs when the Expected profit = 0
- So, $20p - 5(1\text{-}p) = 0$, and therefore $p^* = 1/5$
- By looking at the Normal tables, Z = .842
- Then, $X^* = 584$
- In general, if MR = Marginal Return and ML = Marginal Loss, then $p\ MR - (1\text{-}p)\ ML = 0$
  - $p^* = ML/(MR+ML)$

Using Simulation for stochastic inventory management

What is Simulation ?

Simulation is a model – computer code – "imitating" the operation of a real system in the computer.

It consists of:

a) A set of variables representing the basic features of the real system and

b) A set of logical commands in the computer that modify these features as a function of time in accordance with the rules (logical of physical) regulating the real system.

Main Features of a Simulation System

- The capacity to "advance time" through the use of a simulation build-in clock that monitors and the events while stepping up real time
- The capacity of drawing samples through the creation of artificial observations that behave "like" random events in the real system
  - a) Creation of random numbers (independent & uniformly distributed) by the computer (according to an internal algorithm - function)
  - Conversion in the observations' distribution

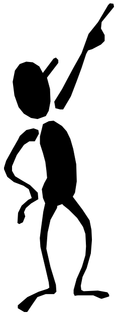
Examples of applications

Very important tool for

- Service management - Analysis of queuing systems
- Business Process Reengineering
- Strategic planning
- Financial planning
- Industrial design (e.g. chemical plants)
- Short term production planning
- Quality and reliability control,
- Training – business games, etc

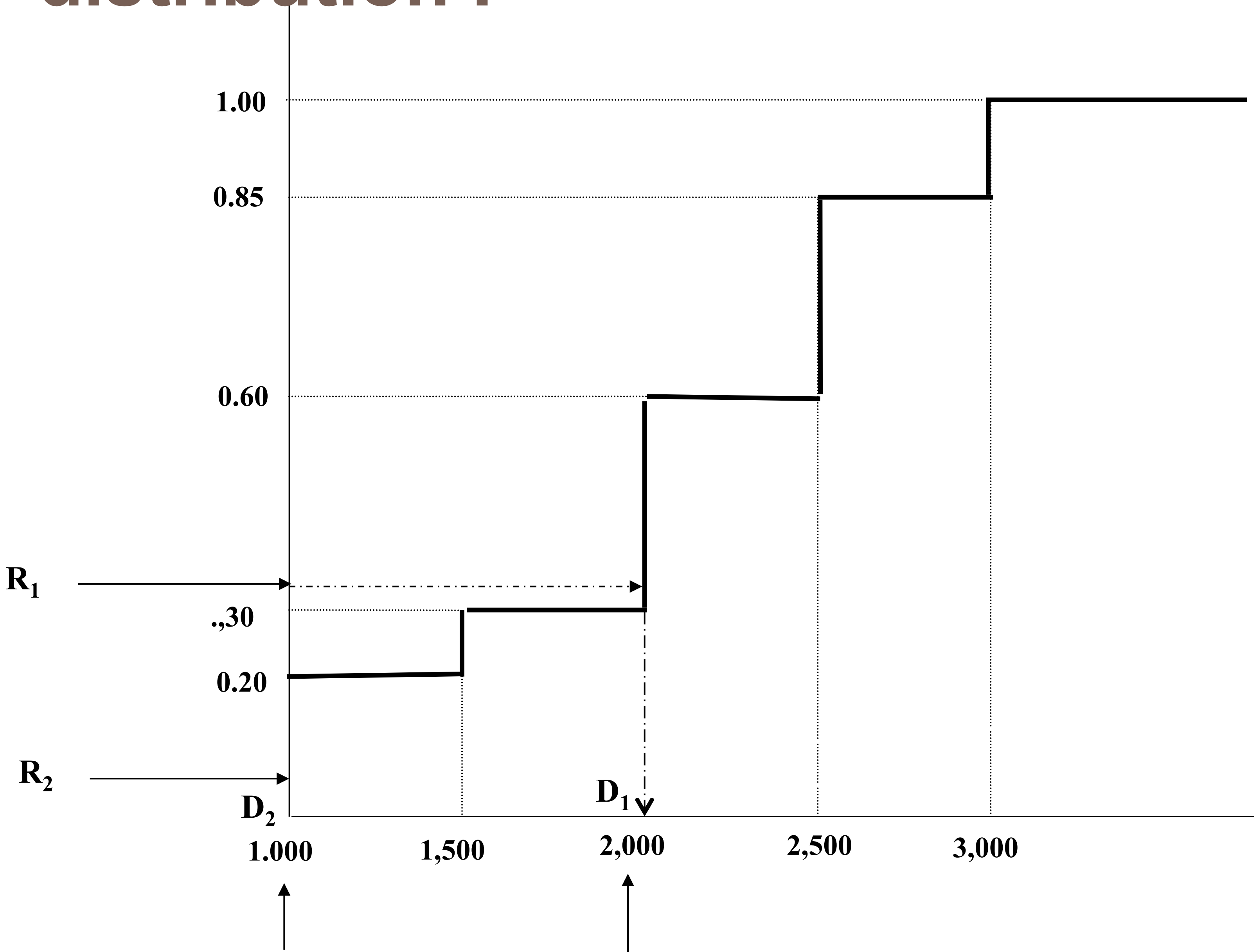
Creating random numbers according to a probability distribution

Let us suppose that the weekly demand of a product can take the following values with the respective probabilities:



Demand (D)	Probability of Demand P(D)	Cumulative Distribution of Demand F(D)
1,000	0.20	0.20
1,500	0.10	0.30
2,000	0.30	0.60
2,500	0.25	0.85
3,000	0.15	1.00

The cumulative probability distribution F



Example

Create a "demand series" corresponding to the random numbers generated

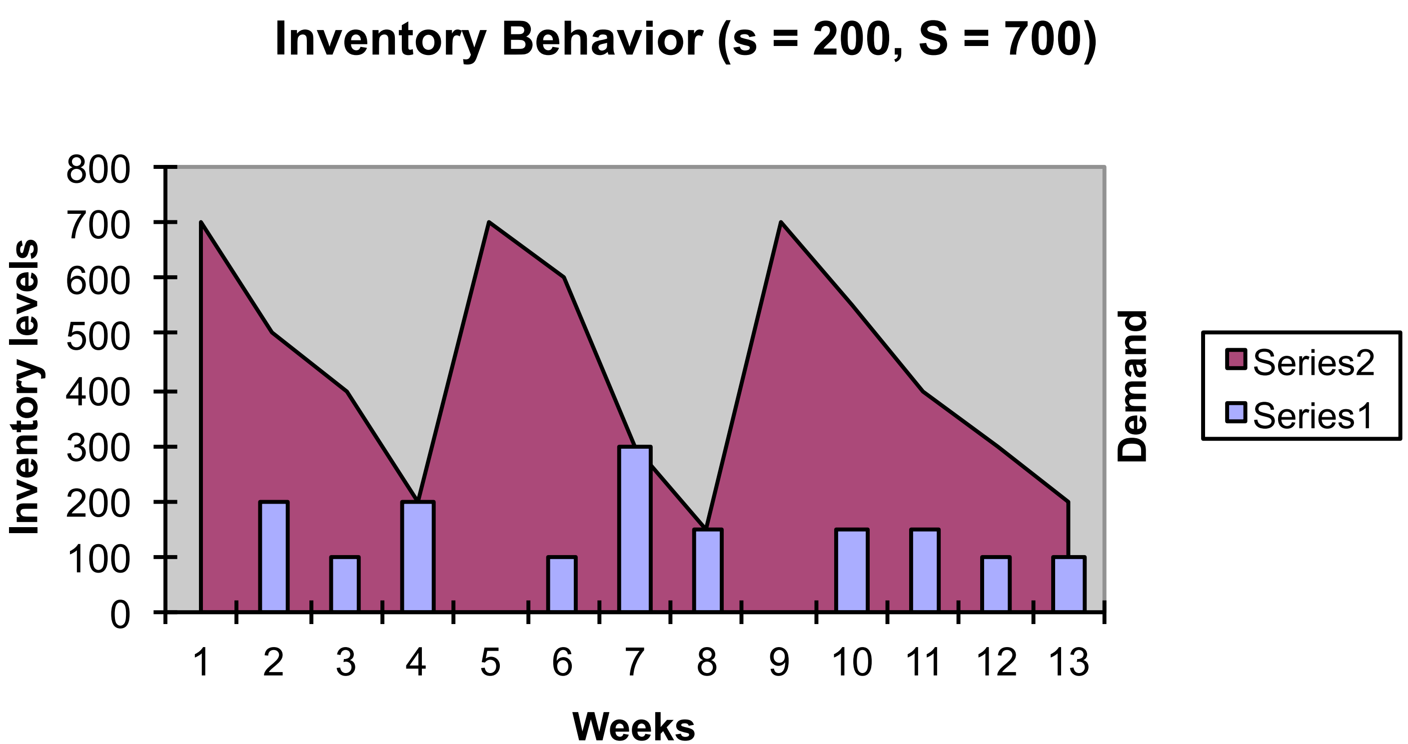
Remember, the Cumulative Distribution F

Demand (D)	Cumulative Distribution of Demand F(D)
1,000	0.20
1,500	0.30
2,000	0.60
2,500	0.85
3,000	1.00

Week	Unif. Random Number	Week's Demand
1	32	2,000
2	8	1,000
3	46	2,000
4	92	3,000
5	69	2,500
6	71	2,500
7	29	1,500
8	46	2,000
9	80	2,500
10	14	1,000

Using Simulation to define an Inventory Policy

- Assume that a company is interested to implement an (s, S) ordering policy. Determine values of s and S:
  - s = Safety stock S = Order-up-to quantity



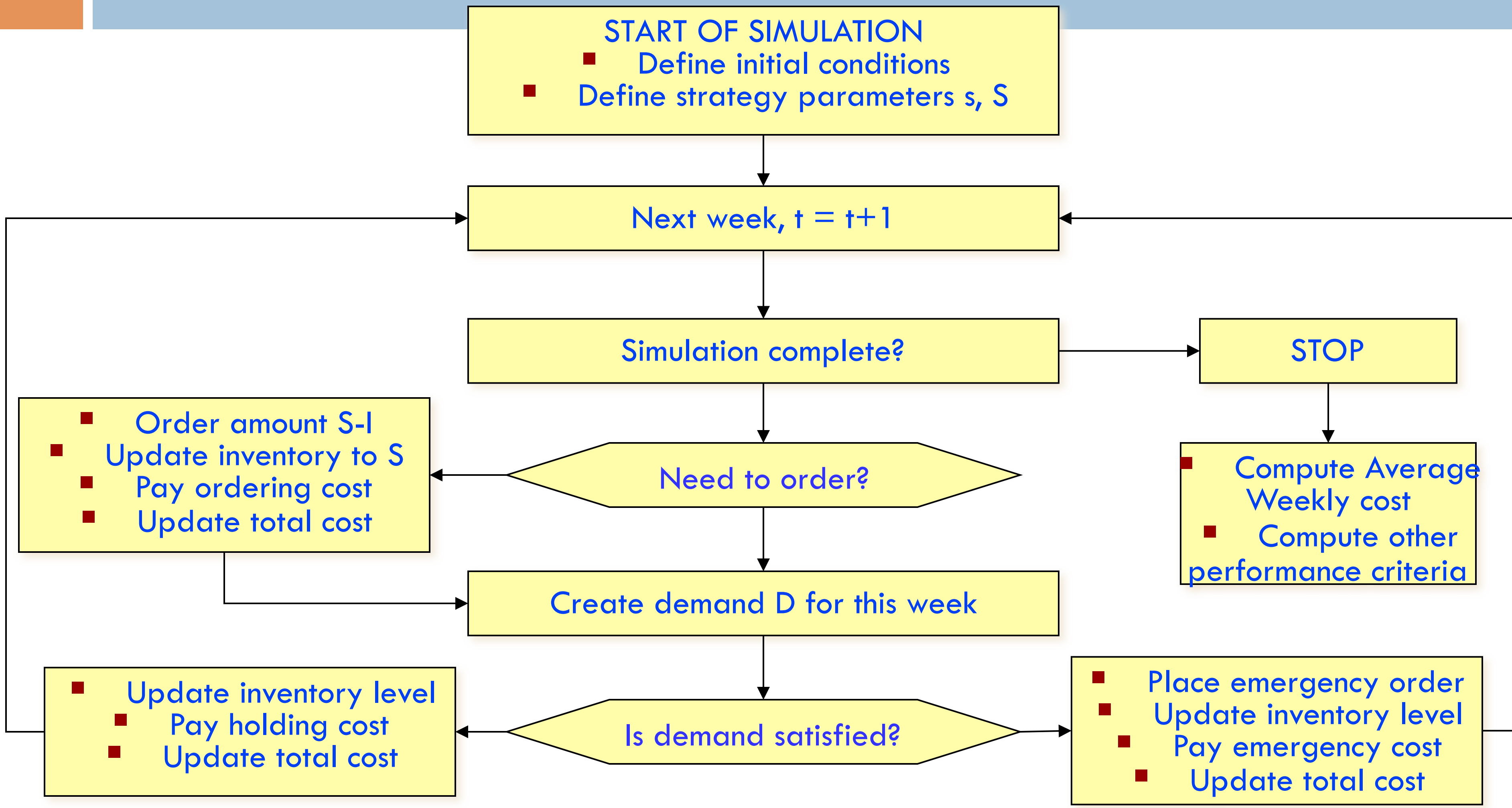
Application to our problem

Demand (D)	Probability of Demand P(D)	Cumulative Distribution of Demand F(D)
1,000	0.20	0.20
1,500	0.10	0.30
2,000	0.30	0.60
2,500	0.25	0.85
3,000	0.15	1.00

Key assumptions:

- When < the safety stock s, order up to the reorder point S
- When short, make emergency order for quantity short
- Normal ordering cost = 200 + 10 • quantity
- Emergency ordering cost = 500 + 15 • quantity
- Leftover inventory cost = 3 • quantity

Flow chart


START OF SIMULATION
Define initial conditions
Define strategy parameters s, S
Next week, t = t+1
Simulation complete?
STOP
Compute Average Weekly cost
Compute other performance criteria
Need to order?
Order amount S-I
Update inventory to S
Pay ordering cost
Update total cost
Create demand D for this week
Is demand satisfied?
Update inventory level
Pay holding cost
Update total cost
Place emergency order
Update inventory level
Pay emergency cost
Update total cost

Manual Simulation of Inventory System

- Assume: s = 1,500 and S = 2,500

Week	Starting Inventory	Need to order?	Size of order	Available inventory	Week's Demand	Emergency order?	Size of emerg. order	Ending Inventory	Weekly Cost
1	2,000	no	0	2,000	2,000	no	0	0	0
2	0	yes	2,500	2,500	2,000	no	0	500	26,700
3	500	yes	2,000	2,500	3,000	yes	500	0	30,500
4	0	yes	2,500	2,500	1,000	no	0	1,500	29,700
5	1,500	yes	1,000	2,500	2,500	no	0	0	10,200
6	0	yes	2,500	2,500	2,500	no	0	0	25,200
7	0	yes	2,500	2,500	3,000	yes	500	0	38,000
8	0	yes	2,500	2,500	1,500	no	0	1,000	28,200
9	1,000	yes	1,500	2,500	2,000	no	0	500	16,700
10	500	yes	2,000	2,500	2,500	no	0	0	20,200
Average	389	always	2,111	2,500	2,200	20%	500	389	25,044