Imagine …

- A limitless & free world
  - In which manufacturing, transportation, warehousing, information sharing … are ALL …
  - Without capacity constraint, without time delays, without costs!
- In this world, we would not need to do any planning to anticipate the demand
- This being not the case, we need to plan in order to be able to satisfy the demand
  - Answer questions that relate to the required capacity, production levels, outsource or not, inventory levels, etc

In today's world …

- Building and having the capacity needed has a cost and lead times are often long
- Aggregate planning:
  - Process by which a company determines planned levels of capacity, production, subcontracting, inventory, stockouts, and pricing over a specified time horizon
  - Goal is to build a plan that will satisfy demand & maximize profit
  - Decisions made at a product family (not SKU) level
  - Time frame of 3 to 18 months: too early to plan production at SKU level, but too late to arrange for additional capacity
  - How can a firm best use the facilities it has?

Role of Aggregate Planning in a Supply Chain

- Specify operational parameters over the time horizon
  - Production rate (unit per period)
  - Subcontracting: 1. yes / no? 2. Subcontracted capacity over the planning horizon?
  - Workforce (number of workers or capacity units needed)
  - Backlog (demand not satisfied for the planning horizon)
  - Overtime
  - Inventory on hand (per period)
  - Machine capacity level
- All supply chain stages should work together on an aggregate plan that will optimize supply chain performance

The Aggregate Planning Problem

- Given the demand forecast for each period in the planning horizon, determine the production level, inventory level, and the capacity level for each period that maximizes the firm's (supply chain's) profit over the planning horizon
- Specify the planning horizon (typically 3-18 months)
- Specify the duration of each period
- Specify key information required to develop an aggregate plan

Information Needed for an Aggregate Plan

- Aggregate demand forecast F_t for each Period t over T periods
- Production costs
  - Labor costs, regular time ($/hr) and overtime ($/hr)
  - Subcontracting costs ($/hr or $/unit)
  - Cost of changing capacity – hiring or layoff ($/worker), adding or reducing machine capacity ($/machine)
- Labor/machine hours required per unit
- Inventory holding cost ($/unit/period)
- Stockout or backlog cost ($/unit/period)
- Constraints – overtime, layoffs, capital available, stockouts, backlogs, from suppliers

Outputs of Aggregate Plan

- Production quantity from regular time, overtime, and subcontracted time, # of workers needed per category, suppliers purchase levels
- Inventory held; warehousing space needed to store it, and working capital required
- Backlog/stockout quantity – used to determine customer service levels
- Workforce hired / laid off – watch out early for labor issues
- Machine capacity increase/decrease – determine new equipment for purchase, or available idle to sell

- A poor aggregate plan can result in lost sales, lost profits, excess inventory, or excess capacity

Identifying Aggregate Units of Production

- Aggregate unit should be identified in a way that the resulting production schedule can be accomplished in practice
- Focus on the bottlenecks when selecting the aggregate unit and identifying capacity and production times
- Account for activities such as setups and maintenance, that eat up capacity and time, but do not result in any production.

The Red Tomato Tools Company

Family	Material Cost/ Unit ($)	Revenue / Unit ($)	Setup Time/Batch (hour)	Average Batch Size	Production Time/ Unit (hour)	Net Production Time/Unit (hour)	Percentage Share of Units Sold
A	15	54	8	50	5.60	5.76	10
B	7	30	6	150	3.00	3.04	25
C	9	39	8	100	3.80	3.88	20
D	12	49	10	50	4.80	5.00	10
E	9	36	6	100	3.60	3.66	20
F	13	48	5	75	4.30	4.37	15

Some simple metrics

- Net production time / unit =

  $= 5.60 + 8/50 = 5.76$hrs

- Weighted average approach

  Material cost per aggregate unit

  $= 15 \times 0.10 + 7 \times 0.25 + 9 \times 0.20$

  $+ 12 \times 0.10 + 9 \times 0.20 + 13 \times 0.15$

  $= \$10$

- Similarly

  Revenue per aggregate unit = $40

  Net production time per aggregate unit = 4.00 hours

Aggregate Planning Strategies

- Trade-off between capacity, inventory, backlog/lost sales: to lower inventory cost, increase capacity cost, or delay delivery to customer

- **Chase** strategy – using capacity as the lever
- **Time flexibility** from workforce or capacity strategy – using utilization as the lever
- **Level strategy** – using inventory as the lever
- Tailored or hybrid strategy – a combination of strategies

Chase Strategy

- Production rate is synchronized with demand rate by varying machine capacity or hiring/laying off employees as demand rate varies
- Vary machine capacity or hire and lay off workers as demand varies
- Often difficult to vary capacity and workforce on short notice
- Expensive if cost of varying capacity is high
- Negative effect on workforce morale
- Results in low levels of inventory
- Used when inventory holding costs are high and costs of changing capacity are low

Time Flexibility Strategy

- Use excess machine capacity, if there is one
- Workforce stable, number of hours worked varies
- Use overtime or a flexible work schedule
- Flexible workforce, avoids morale problems
- Low levels of inventory, lower utilization
- Used when inventory holding costs are high and capacity is relatively inexpensive

Level Strategy

- Use inventory as the level
- Stable machine capacity and workforce levels, constant output rate
- Inventory levels fluctuate over time
- Inventories carried over from high to low demand periods
- Better for worker morale
- Large inventories and backlogs may accumulate
- Used when inventory holding and backlog costs are relatively low

Aggregate Planning Using Linear Programming

- Red Tomato Tools
  - Highly seasonal demand
  - Develop a forecast

Month	Demand Forecast
January	1,600
February	3,000
March	3,200
April	3,800
May	2,200
June	2,200

Red Tomato Tools company

- Each tool is sold through retailers for 40$
- Starting inventory of 1,000 tools
- Workforce of 80 employees; 20 working days in each month
- 8 hour per day for each employee + 2 overtime
- Capacity is determined by total labor hours worked
- No limits on subcontracting, inventory and stockouts/backlogs
- All stockouts are backlogged and supplied from next months production
- Inventory costs appear at the end of each month
- Pay $4/hr for regular time; OT has to be $\leq$ 10 hrs/mo

- What is the optimal aggregated plan that allows at the end of June to have at least 500 units of inventory

Costs for Red Tomato Tools

Item	Cost
Material cost	$10/unit
Inventory holding cost	$2/unit/month
Marginal cost of stockout/backlog	$5/unit/month
Hiring and training costs	$300/worker
Layoff cost	$500/worker
Labor hours required	4/unit
Regular time cost	$4/hour (no of units produced or regular time = 40/mo)
Overtime cost	$6/hour
Cost of subcontracting	$30/unit

Decision Variables

For month $t = 1, \ldots, 6$

W_t = Workforce size for month t
H_t = Number of employees hired at the beginning of month t
L_t = Number of employees laid off at the beginning of month t
P_t = Production in month t
I_t = Inventory at the end of month t
S_t = Number of units stocked out or backlogged at the end of month t
C_t = Number of units subcontracted for month t
O_t = Number of overtime hours worked in month t

Objective Function

- Minimize (assume half-year horizon)
  (Regular-time labor cost + Overtime labor cost + Cost of hiring and layoffs + Cost of holding inventory + Cost of stocking out + Cost of subcontracting + Material cost)

$$Min \sum_{t=1}^{6} 640W_t + \sum_{t=1}^{6} 6O_t + \sum_{t=1}^{6} 300H_t + \sum_{t=1}^{6} 500L_t$$

$$+ \sum_{t=1}^{6} 2I_t + \sum_{t=1}^{6} 5S_t + \sum_{t=1}^{6} 10P_t + \sum_{t=1}^{6} 30C_t$$

$4/hr x 8 hrs/day x 20 days

Constraints

1. Workforce, hiring, and layoff constraints

$$W_t = W_{t-1} + H_t - L_t$$

With $W_0 = 80$

2. Capacity constraints

$$P_t \text{ £ } 40W_t + \frac{O_t}{4}$$

All for $t = 1,\ldots, 6$

3. Inventory balance constraints

$$I_{t-1} + P_t + C_t = D_t + S_{t-1} + I_t - S_t$$

With $I_0 = 1{,}000$

4. Overtime limit constraints

$$O_t \text{ £ } 10W_t$$

Optimal Aggregate Plan

Total cost over planning horizon = \$422,660

Revenue over planning horizon = 40 x 16,000 = \$640,000

Period t	No. Hired, H_t	No. Laid Off, L_t	Work-force Size, W_t	Over-time, O_t	Inventory, I_t	Stock -out, S_t	Sub-contract, C_t	Total Production, P_t
0	0	0	80	0	1,000	0	0	
1	0	16	64	0	1,960	0	0	2,560
2	0	0	64	0	1,520	0	0	2,560
3	0	0	64	0	880	0	0	2,560
4	0	0	64	0	0	220	140	2,560
5	0	0	64	0	140	0	0	2,560
6	0	0	64	0	500	0	0	2,560

Calculating Averages

$$\text{Average inventory} = \frac{(I_0 + I_T)/2 + \left(\sum_{t=1}^{T-1} I_t\right)}{T}$$

Average flow time = Average inventory / throughput
(Little' law)

$$\begin{array}{c}\text{Average time}\\ \text{in inventory}\end{array} = \left[\frac{(I_0 + I_T)/2 + \left(\sum_{t=1}^{T-1} I_t\right)}{T}\right] \Bigg/ \left[\frac{\left(\sum_{t=1}^{T-1} D_t\right)}{T}\right]$$

Average inventory

Average demand

Calculating Averages

$$\text{Average seasonal inventory} = \frac{(I_0 + I_6)/2 + \left(\sum_{t=1}^{5} I_t\right)}{T} = \frac{5{,}250}{6} = 875$$

$$\text{Average flow time} = \frac{875}{2{,}667} = 0.33 = 0.33\ \text{months}$$

Little's law

Impact of higher demand variability

Month	Demand Forecast
January	1,000
February	3,000
March	3,800
April	4,800
May	2,000
June	1,400

$$\text{Average seasonal inventory} = \frac{(I_0 + I_T)/2 + \left(\sum_{t=1}^{T-1} I_t\right)}{T} = \frac{6{,}310}{6} = 1{,}052$$

Optimal Aggregate Plan

Total cost over planning horizon = $433,080

Period, t	No. Hired, H_t	No. Laid Off, L_t	Workforce Size, W_t	Overtime, O_t	Inventory, I_t	Stockout, S_t	Subcontract, C_t	Total Production, P_t
0	0	0	80	0	1,000	0	0	
1	0	16	64	0	2,560	0	0	2,560
2	0	0	64	0	2,120	0	0	2,560
3	0	0	64	0	880	0	140	2,560
4	0	0	64	0	0	1,220	0	2,560
5	0	0	64	0	0	660	0	2,560
6	0	0	64	0	500	0	0	2,560

$$\text{Average flow time} = \frac{1{,}052}{2{,}667} = 0.39 \text{ months}$$

Red Tomato Tools

- Lower hiring and layoff costs (by 50$ each)

Total cost over planning horizon = \$412,780

$$\text{Average seasonal inventory} = \frac{(I_0 + I_T)/2 + \left(\sum_{t=1}^{T-1} I_t\right)}{T} = \frac{2,450}{6} = 408$$

$$\text{Average flow time} = \frac{408}{2,667} = 0.15 \text{ months}$$

Red Tomato Tools

Period, t	No. Hired, H_t	No. Laid Off, L_t	Workforce Size, W_t	Overtime, O_t	Inventory, I_t	Stockout, S_t	Subcontract, C_t	Total Production, P_t
0	0	0	80	0	1,000	0	0	
1	0	35	45	0	1,200	0	0	1800
2	0	0	45	0	0	0	0	1800
3	42	0	87	0	280	0	0	3,480
4	0	0	87	0	0	20	20	3,480
5	0	26	61	0	220	0	0	2,440
6	0	0	62	0	500	0	0	2,480

Forecast Error in Aggregate Plans

- Forecast errors must be considered
- *Safety inventory*
- *Safety capacity*
  - Use overtime as a form of safety capacity
  - Carry extra workforce permanently as a form of safety capacity
  - Use subcontractors as a form of safety capacity
  - Build and carry extra inventories as a form of safety inventory
  - Purchase capacity or product from an open or spot market as a form of safety capacity